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LSM2: THE BANKING AND DISTRIBUTION SECTORS IN A DSGE MODEL FOR LUXEMBOURG



LSM2: THE BANKING AND DISTRIBUTION SECTORS IN A DSGE MODEL FOR LUXEMBOURG The "Perspectives de Politique Économique" series includes reports, studies, research results or summaries of conferences commanded by or carried out by employees of the Ministry of Economy and Foreign Trade or by experts of associated institutions.

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LSM2: the banking and distribution sectors in a DSGE Model for Luxembourg

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June 2012

Abstract

The recent crisis has emphasized the role of financial - macroeconomic interactions, and international trade in goods and services, in the transmission of the shocks. Both phenomena, closely related to the higher degree of globalization, are very relevant for small open economies, and particularly so when a large share of the economy relies on financial and distribution services. Hence, in this paper we propose to incorporate the banking and distribution sectors into a medium scale DSGE model of a small open economy. The resulting model is then calibrated to match the specific characteristics of the Luxembourg economy, but we believe that the results are also of more general interest for studying the reaction of small open economies to real and financial shocks.

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1 Introduction

The recent crisis has emphasized the role of financial - macroeconomic interactions, and international trade in goods and services, in the transmission of the shocks. Both phenomena, closely related to the higher degree of globalization, are very relevant for small open economies, and particularly so when a large share of the economy relies on financial and distribution services. Hence, in this paper we propose to incorporate the banking and trade sectors into a medium scale Dynamic Stochastic General Equilibrium (DSGE) model of a small open economy. The resulting model is then calibrated to match the specific characteristics of the Luxembourg economy, and used to assess the consequences of a series of policies targeting the financial sectors. We believe that the results are also of more general interest for studying the reaction of small open economies to such policies.

The starting point is LSM (Luxembourg Structural Model), the DSGE model for Luxembourg developed in Deak, Fontagne, Maffezzoli and Marcellino (2011). We introduce in this model a distribution sector, and a financial sector comprising two tiers: domestic and international. It requires to properly modify all the other sectors of the original model.

There are five types of agents: Households, Government, Firms, Banks and Unions. Households have finite lives, with a set of overlapping generations with different features in each time period, and each household maximizes an intertemporal utility function subject to a budget constraint, determining the optimal amount of consumption and financial assets. The individual households' decisions are then aggregated to determine aggregate quantities.

The Government collects taxes on the returns from assets and on labour income. The tax receipts are used to finance expenditures, which are made up of unemployment benefits, other transfers to resident and non-resident population, and public investment. When the receipts are less (more) than the expenditures there is a deficit (surplus), whose evolution over time, combined with that of interest rates, determines the level of the public debt, which is financed with the emission of government bonds.

The interest rate is taken as exogenous, in line with the small open economy assumption. However, following Schmitt-Grohé and Uribe (2004), we assume the existence of a debt-elastic interest-rate premium, i.e. an interest rate that is increasing in the country's net foreign debt.

Assets are made up of government bonds, foreign assets and claims to physical capital. These three types of assets are perfect substitutes in the household's portfolio, and earn in equilibrium the same (exogenous) real rate of return. Investment in physical capital is determined by maximizing the cash flow from investing in physical capital, conditional on the law of motion of physical capital. Households are also in charge of investment, and therefore they supply capital (and labour).

Firms produce intermediate and final goods. In the (differentiated) intermediate goods sector firms operate under monopolistic competition, using a Nested CES production function with capital and two different types of labour as inputs. The different types of labour are introduced to mimic the dual labour market in Luxembourg and several other small open economies, and represent resident and non-resident workers. The firms choose the optimal demand of capital and of each type of labour by maximizing profits, subject to the production function constraint, taking wages and the cost of capital as given. The cost of capital is determined endogenously in order to match the demand and supply of capital. In the final goods sector, firms operate under perfect competition, using a Nested CES production function with intermediate goods only as inputs, possibly with increasing returns to variety. Public investment increases productivity, in addition to exogenous technical progress.

The wages are determined by the interaction between the firms and the unions that represent the workers (the so-called "right to manage" model). Given the resulting wages, labour demand is determined, and it is assumed that for the current wages the supply of each type of labour adjusts to meet demand.

We introduce two banking sectors, one with domestic banks and one with international banks. In the domestic banking sector there exists a representative, competitive bank that intermediates loans between households and firms, producing financial services using capital, labor and reserves. The bank is owned by the domestic households. As in Christiano and Eichenbaum (1995) and Christiano, Motto, and Rostagno (2008), firms (and banks themselves) will need to finance a given fraction of their factor expenditure in advance, before revenues from sales can be cashed in, i.e. firms will need short-term working capital.

In the international banking sector, there exists a representative, competitive bank that intermediates loans between foreign households and foreign firms, producing financial services using capital, labor and reserves. The bank is owned by the local households. To finance the foreign demand for loans, the bank issues demand deposit liabilities to foreign households. We assume that the supply of foreign demand deposits is exogenous, and depends positively on the interest rate paid by the international bank.

We also introduce the distribution sector, where importers buy foreign varieties at a give exogenous price (including tariffs). Foreign varieties are combined with physical capital and labor to produce distribution services, which are then sold to domestic final producers and/or re-exported. Foreign demand for re-exported goods will be modeled as exogenous, in line with the usual small open economy assumption.

We will refer to the resulting model as LSM2.¹ Due to the complexity of LSM2 and the availability of only 15 years of quarterly observations for Luxembourg, the model cannot be estimated and is therefore fully calibrated, matching several of the key characteristics of the Luxembourg economy.

Finally, the calibrated model is used to assess the effects of a set of policy targeting the real and financial sectors.

The paper is structured as follows. In Section 2 we describe in details the different sectors of LSM2. In Section 3 we briefly discuss the equilibrium conditions, with full details provided in Appendix A. In Section 4 we discuss the calibration of LSM2, with full details in Appendix B. In Section 5 we use LSM2 to analyze the effects of increasing competition in the Luxembourg product and labour markets, comparing the results with those obtained with LSM. In Section 6 we conduct a set of experiments related to modifications in the banking and re-export sectors. In Section 7 we summarize the main results and conclude.

¹The three other main macroeconometric models for Luxembourg are: the STATEC model Modux (Adam (2004, 2007)), the model of the Banque Central du Luxembourg (Guarda (2005)), and the STATEC multi-sector model LuxMod (STATEC (2006)). Each model was developed for specific purposes and none belongs to the NOEM-DSGE class. This is the distinctive feature of LSM2, which makes it more suitable than the other models for policy simulations, though perhaps less adapted to other uses, such as short and medium-term forecasting.

2 The structure of LSM2

In this section we provide a detailed description of the main components of the model. Specifically, we discuss, in turn, households, financial assets, firms and unions, the domestic and international banks, and the government. The derivations for the households, financial assets, firms, unions and the government are similar to those in LSM, with proper changes to take into consideration the inclusion of the banking and distribution sectors.

2.1 Households

2.1.1 The household's problem at the cohort level

Following Blanchard (1985), in period t, the representative consumer of generation z maximizes her expected lifetime utility:

$$\mathbf{u}_{z,t} = \sum_{s=t}^{\infty} \left(\varphi\beta\right)^{s-t} u\left(x_{z,s}\right) \tag{1}$$

where $\varphi \in (0, 1)$ represents the constant *probability of survival*, i.e. the share of individuals that survive in each period, β the subjective discount factor, $x_{z,t} \equiv \{c_{z,t}, d_{z,t}, m_{z,t}/p_t\}$ with c_t denoting non-durable consumption (from now on, consumption *tout court*), d_t the end-of-period desired stock of durable consumption goods (from now on, durables), $m_{z,t}$ the real money holdings kept for transaction purposes, and p_t the price of non-durable consumption. The utility function, $u(x_{z,t})$, is of the constant relative risk aversion (CRRA) type:

$$u\left(x_{z,t}\right) \equiv \frac{\left\{ \left[\phi_C c_{z,t}^{\upsilon} + \phi_D d_{z,t}^{\upsilon} + \phi_M \left(\frac{m_{z,t}}{p_t}\right)^{\upsilon} \right]^{\frac{1}{\upsilon}} \right\}^{1-\sigma} - 1}{1-\sigma}.$$
(2)

where $\phi_C + \phi_D + \phi_M = 1$.

There is a fixed individual endowment of money, \bar{m} , that can be used as cash for transaction purposes, or invested into demand deposits. Demand deposits are liquidated before the end of the period, and pay an interest rate i_t^D . Hence, the period-by-period budget constraint for the representative agent in generation z can be written as:

$$a_{z,t} = \frac{R_t}{\varphi} a_{z,t-1} + (1 - \tau_K) i_t^D \left(\bar{m} - m_{z,t}\right) + \omega_t + - (1 + \tau_C) p_t \left[c_{z,t} + \varkappa_t^d \underbrace{\left(d_{z,t} - \frac{1 - \delta_D}{\varphi} d_{z,t-1} \right)}_{\text{Investment in durables}} \right], \quad (3)$$

where:

$$R_t \equiv 1 + (1 - \tau_K) i_t. \tag{4}$$

The variables are defined as follows: a_t is the end-of-period asset stock, R_t is gross rate of return common across assets, τ_K is the tax rate on financial asset returns, i_t the *exogenous* (small open economy assumption) interest rate, i_t^D the endogenous interest rate on demand deposits, ω_t is current non-financial income, p_t is the price of the final good, τ_C is the tax rate on consumption, δ^D is the depreciation rate of durables, and \varkappa_t^d is an exogenous shock to the relative price for durables. Note that we are assuming that the final consumption good can be transformed into durables at a rate \varkappa_t^d . Furthermore, note that $a_{t,t-1} = 0$, for $t \ge z$, meaning that new generations have no endowments.

Following Schmitt-Grohe and Uribe (2004), we assume the existence of a debt-elastic interest-rate premium, i.e. an interest rate that is increasing in the country's net foreign debt:

$$i_t = \bar{\imath} + \xi_i \left[\exp\left(\bar{F} - \frac{F_t}{GDP_t}\right) - 1 \right] + \varepsilon_{it}$$
(5)

where F_t represents the country's net foreign asset position, \bar{i} the long-run, constant, and exogenous interest rate if the country runs its steady-state net foreign asset position (\bar{F}), and ε_{it} an interest-rate shock.

Current non-financial income is defined as:

$$\omega_t \equiv \underbrace{(1 - \tau_L) \left[w_{1,t} h_{1,t} + \bar{w}_{1,t} \left(1 - h_{1,t} \right) \right]}_{\text{Labor income}} + (1 - \tau_K) \pi_t + tr_t, \tag{6}$$

where $h_{1,t}$ is the employment rate of resident workers (at the individual level, the unemployment rate can be interpreted as the probability of being unemployed), $w_{1,t}$ their wage rate², τ_L the tax rate on labour related income, $\bar{w}_{1,t}$ the unemployment benefits for resident former workers, π_t the exogenous, individual share of total firm profits, and tr_t the net government transfer. Note that the expression for labour income reflects the assumption of perfect unemployment insurance, and the existence of two types of labour, resident and non-resident.

Notice that, even if the life expectancy of the consumer decreases exponentially, she could still live for an infinite number of periods. Therefore, it is important to impose as an additional constraint the no-Ponzi game condition:

$$\lim_{T \to \infty} \prod_{s=0}^{T} \varphi \frac{a_{z,t+s}}{R_{t+s}} = 0, \tag{7}$$

which prevents overborrowing.

The intertemporal budget constraint, obtained by iterating on (3) and imposing the NPG condition in (7), is:

$$(1+\tau_C)\sum_{s=t}^{\infty} \mathcal{R}_{t,s} p_s \left[c_{z,s} + \varkappa_s^d \left(d_{z,s} - \frac{1-\delta_D}{\varphi} d_{z,s-1} \right) \right] = \frac{R_t}{\varphi} a_{z,t-1} + \sum_{s=t}^{\infty} \mathcal{R}_{t,s} \left[(1-\tau_K) \, i_s^D \left(\bar{m} - m_{z,s} \right) + \omega_s \right], \quad (8)$$

where $\mathbf{R}_{t,t} \equiv 1$ and, for $s \ge t+1$,

$$\mathbf{R}_{t,s} \equiv \prod_{j=t+1}^{s} \frac{\varphi}{R_j}.$$
(9)

The households maximizes the objective function (1) subject to (3) and (7). The Lagrangian can be $\frac{1}{2^{2}}$ More precisely, for the sake of notational simplicity, w_{1t} is the weighted average of sectoral wages, weighted with the corresponding employment shares (see eq. 319, p. 56).

written as:

$$L_{z,t} = \sum_{s=t}^{\infty} (\varphi \beta)^{s-t} \left\{ u \left(x_{z,s} \right) + \lambda_{z,s} \left[\frac{R_t}{\varphi} a_{z,s-1} + (1 - \tau_K) \, i_s^D \left(\bar{m} - m_{z,s} \right) + (1 + \tau_C) \, p_s \varkappa_s^d \frac{1 - \delta_D}{\varphi} d_{z,s-1} + \omega_s - (1 + \tau_C) \, p_s \varkappa_s^d d_{z,s-1} + \omega_s - (1 + \tau_C) \, p_s \varkappa_s^d d_{z,s-1} \right\} \right\}, \quad (10)$$

and the resulting first-order conditions (w.r.t. $c_{z,t},\,a_{z,t},\,d_{z,t},\,m_{z,t})$ are:

$$u_c(x_{z,t}) = (1 + \tau_C) p_t \lambda_{z,t}, \qquad (11)$$

$$\lambda_{z,t+1}\beta R_{t+1} = \lambda_{z,t},\tag{12}$$

$$u_d(x_{z,t}) + \beta \lambda_{z,t+1} (1 + \tau_C) p_{t+1} \varkappa_{t+1}^d (1 - \delta_D) = (1 + \tau_C) p_t \varkappa_t^d \lambda_{z,t},$$
(13)

$$u_m(x_{z,t}) = \lambda_{z,s} \left(1 - \tau_K\right) i_t^D,\tag{14}$$

where:

$$u_{c}(x_{z,t}) = \left[\phi_{C}c_{z,t}^{\upsilon} + \phi_{D}d_{z,t}^{\upsilon} + \phi_{M}\left(\frac{m_{z,t}}{p_{t}}\right)^{\upsilon}\right]^{\frac{1-\upsilon-\sigma}{\upsilon}}\phi_{C}c_{z,t}^{\upsilon-1},$$
(15)

$$u_{d}(x_{z,t}) = \left[\phi_{C}c_{z,t}^{\upsilon} + \phi_{D}d_{z,t}^{\upsilon} + \phi_{M}\left(\frac{m_{z,t}}{p_{t}}\right)^{\upsilon}\right]^{-\frac{\upsilon}{\upsilon}} \phi_{D}d_{z,t}^{\upsilon-1},$$
(16)

$$u_m(x_{z,t}) = \left[\phi_C c_{z,t}^{\nu} + \phi_D d_{z,t}^{\nu} + \phi_M \left(\frac{m_{z,t}}{p_t}\right)^{\nu}\right]^{\frac{1-\nu-\sigma}{\nu}} \frac{\phi_M}{p_t} \left(\frac{m_{z,t}}{p_t}\right)^{\nu-1}.$$
 (17)

Substitution of (11) into (12), (13), and (14) yields the two Euler equations:

$$u_{c}(x_{z,t+1})\beta R_{t+1}\frac{p_{t}}{p_{t+1}} = u_{c}(x_{z,t}), \qquad (18)$$

$$u_d(x_{z,t}) + \beta (1 - \delta_D) \varkappa_{t+1}^d u_c(x_{z,t+1}) = \varkappa_t^d u_c(x_{z,t}),$$
(19)

and an intratemporal condition:

$$u_m(x_{z,t}) = u_c(x_{z,t}) \frac{(1 - \tau_K) i_t^D}{(1 + \tau_C) p_t}.$$
(20)

Combining (20) with (15) and (17) gets:

$$m_{z,t} = \iota_t c_{z,t}.\tag{21}$$

where:

$$\iota_t \equiv \left(\frac{\phi_C}{\phi_M} \frac{1 - \tau_K}{1 + \tau_C} i_t^D\right)^{\frac{1}{\nu - 1}} p_t.$$

$$(22)$$

Combining (18)-(19) and (15), we can express optimal durables in terms of optimal consumption as:

$$d_{z,t} = \xi_t c_{z,t},\tag{23}$$

where:

$$\xi_t \equiv \left\{ \frac{\phi_C}{\phi_D} \left[\varkappa_t^d - \varkappa_{t+1}^d \frac{1 - \delta_D}{R_{t+1} \frac{p_t}{p_{t+1}}} \right] \right\}^{\frac{1}{\nu - 1}}.$$
(24)

For optimal consumption, from (18) we obtain:

$$c_{z,t+1} = \mathcal{E}_{t+1}c_{z,t},\tag{25}$$

where:

$$\mathcal{E}_{t+1} \equiv \left\{ \left[\frac{\phi_C + \phi_D \xi_{t+1}^{\upsilon} + \phi_M \iota_{t+1}^{\upsilon}}{\phi_C + \phi_D \xi_t^{\upsilon} + \phi_M \iota_t^{\upsilon}} \right]^{\frac{1-\upsilon-\sigma}{\upsilon}} \beta R_{t+1} \frac{p_t}{p_{t+1}} \right\}^{\frac{1}{\sigma}}.$$
 (26)

Equations (21), (23), and (25) imply that:

$$\sum_{s=t}^{\infty} \mathbf{R}_{t,s} \left\{ (1+\tau_C) \, p_s \left[c_{z,s} + \varkappa_t^d \left(d_{z,s} - \frac{1-\delta_D}{\varphi} d_{z,s-1} \right) \right] + (1-\tau_K) \, i_s^D m_{z,s} \right\} = \sum_{s=t}^{\infty} \mathbf{R}_{t,s} c_{z,s} \left\{ (1+\tau_C) \, p_s \left[1 + \varkappa_s^d \left(\xi_s - \frac{1-\delta_D}{\varphi} \frac{\xi_{s-1}}{\mathcal{E}_s} \right) + (1-\tau_K) \, i_s^D \iota_s \right] \right\} = \sum_{s=t}^{\infty} \mathbf{R}_{t,s} c_{z,s} \mathcal{Z}_s = c_{z,t} \mathcal{Z}_t + \frac{\varphi}{R_{t+1}} c_{z,t+1} \mathcal{Z}_{t+1} + \prod_{j=1}^2 \frac{\varphi}{R_{t+j}} c_{z,t+2} \mathcal{Z}_{t+2} + \dots = c_{z,t} \mathcal{Z}_t + \frac{\varphi \mathcal{E}_{t+1}}{R_{t+1}} c_{z,t} \mathcal{Z}_{t+1} + \prod_{j=1}^2 \frac{\varphi \mathcal{E}_{t+j}}{R_{t+j}} c_{z,t} \mathcal{Z}_{t+2} + \dots = \zeta_t c_{z,t}, \quad (27)$$

where:

$$\zeta_t \equiv \sum_{j=0}^{\infty} \mathcal{Z}_{t+j} \varphi^j \prod_{s=1}^j \frac{\mathcal{E}_{t+s}}{R_{t+s}},\tag{28}$$

and:

$$\mathcal{Z}_t \equiv (1+\tau_C) p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1-\delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right] + (1-\tau_K) i_t^D \iota_t.$$
⁽²⁹⁾

Note that $Z_t c_{z,t}$ represents the total value of current consumption, net investment in durables, and cash holdings for generation z in period t, being the demand for durables and cash related to the demand for consumption goods via (21) and (23). The term $\zeta_t c_{z,t}$, instead, represents the total *discounted* flow of future consumption, net investment in durables, and cash holdings.

Note also that ζ_t can be defined recursively as:

$$\zeta_t = \mathcal{Z}_t + \mathcal{E}_{t+1} \frac{\varphi}{R_{t+1}} \zeta_{t+1}.$$
(30)

Multiplying both sides by $c_{z,t}$, we can easily provide a simple interpretation:

$$\zeta_t c_{z,t} = \mathcal{Z}_t c_{z,t} + \frac{\varphi}{R_{t+1}} \zeta_{t+1} \underbrace{(\mathcal{E}_{t+1} c_{z,t})}_{c_{z,t+1}}.$$

The discounted flow of future "consumption" $\zeta_t c_{z,t}$ (i.e. consumption plus net investment in durables plus cash) equals the current value of "consumption," $\mathcal{Z}_t c_{z,t}$, plus the discounted value of the one-period-ahead flow, $\zeta_{t+1} c_{z,t+1}$.

Combining (8) and (27), we can therefore write optimal current consumption as:

$$c_{z,t} = \zeta_t^{-1} \left(\frac{R_t}{\varphi} a_{z,t-1} + h_t^w \right), \tag{31}$$

where:

$$h_t^w \equiv \sum_{s=t}^{\infty} \mathcal{R}_{t,s} \left[(1 - \tau_K) \, i_s^D \bar{m} + \omega_s \right], \tag{32}$$

represents human wealth plus the current value of the individual money stock.

2.1.2 Aggregation

Let us assume that the size of each new-born generation is z_t , where $z_t = \eta^t z_{-\infty}$ and $z_{-\infty}$ is normalized to one. Then, the total population at any date t, Z_t , is equal to:

$$Z_{t} = \underbrace{z_{t}}_{\text{Generation t}} + \underbrace{\varphi z_{t-1}}_{\text{Generation t-1}} + \underbrace{\varphi^{2} z_{t-2}}_{\text{Generation t-2}} + \dots = \\ = \eta^{t} \left[1 + \frac{\varphi}{\eta} + \left(\frac{\varphi}{\eta}\right)^{2} + \dots \right] = \eta^{t} \sum_{j=0}^{\infty} \left(\frac{\varphi}{\eta}\right)^{j} = \frac{z_{t}}{1 - \frac{\varphi}{\eta}}, \quad (33)$$

and it is

$$Z_{t+1} = \eta Z_t$$

The expressions for the aggregate variables can be obtained by linear aggregation of those at the cohort level. Let us start with aggregate assets. We have

$$A_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} a_{z_{t-j},t}.$$
(34)

Aggregating the budget constraint in (3) over cohorts, we obtain an equation describing the evolution of aggregate assets:

$$A_t = R_t A_{t-1} + \mathcal{H}_t - \mathcal{Z}_t C_t, \tag{35}$$

where

$$\mathcal{H}_t \equiv \left[(1 - \tau_K) \, i_t^D \bar{m} + \omega_t \right] Z_t,\tag{36}$$

and $Z_t C_t$ represents the total aggregate value of current consumption and net investment in durables. Equation (35) can be considered as the budget constraint at the aggregate level.

Next, let us consider aggregate net human wealth, where cohort-level human wealth, h_t^w , is defined in equation (32). We have:

$$H_t^w \equiv \sum_{j=0}^\infty \varphi^j z_{t-j} h_t^w = h_t^w Z_t.$$
(37)

The evolution of aggregate net human wealth is given by

$$H_{t+1}^{w} = \frac{\eta}{\varphi} R_{t+1} \left(H_t^{w} - \mathcal{H}_t \right), \tag{38}$$

since,

$$H_{t+1}^{w} = Z_{t+1} \sum_{s=t+1}^{\infty} \mathcal{R}_{t+1,s} \left[\omega_s + (1 - \tau_K) \, i_s^D \bar{m} \right] =$$
(39)

$$= Z_{t+1} R_{t,t+1}^{-1} \left\{ \sum_{s=t}^{\infty} R_{t,s} \left[\omega_s + (1 - \tau_K) \, i_s^D \bar{m} \right] - \left[\omega_t + (1 - \tau_K) \, i_t^D \bar{m} \right] \right\} = \frac{\eta}{\varphi} R_{t+1} \left(H_t^w - \mathcal{H}_t \right).$$
(40)

For aggregate consumption, aggregating equation (31) over cohorts yields:

$$C_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} c_{z_{t-j},t} = \zeta_t^{-1} \left(R_t A_{t-1} + H_t^w \right), \tag{41}$$

where aggregate assets, A_t , are defined in (34) and aggregate human wealth, H_t^w , in (37). The evolution of aggregate consumption is governed by the aggregate Euler equation

$$C_{t+1} = \eta \mathcal{E}_{t+1} \left(C_t - \frac{\eta - \varphi}{\eta} \frac{A_t}{\zeta_t - \mathcal{Z}_t} \right).$$
(42)

In order to derive this equation, aggregation of the Euler equations at the cohort level, reported in (25), yields,

$$C_{t+1} = \sum_{j=0}^{\infty} \varphi^j z_{t+1-j} c_{t+1-j,t+1} = z_{t+1} \zeta_{t+1}^{-1} h_{t+1}^w + \varphi \mathcal{E}_{t+1} C_t,$$

where the first term on the right-hand side reflects the future consumption of the new generation that will enter the market in period t + 1 with no financial endowments and only non-financial income. Since

$$z_{t+1}h_{t+1}^w = Z_{t+1}\left(1 - \frac{\varphi}{\eta}\right)h_{t+1}^w = \left(1 - \frac{\varphi}{\eta}\right)H_{t+1}^w,$$

it is

$$C_{t+1} = \varphi \mathcal{E}_{t+1} C_t + \zeta_{t+1}^{-1} \left(1 - \frac{\varphi}{\eta} \right) H_{t+1}^w.$$

$$\tag{43}$$

Furthermore, combining (38), (41), and (35) gets:

$$H_{t+1}^{w} = \frac{\eta}{\varphi} R_{t+1} \left[\left(\zeta_t - \mathcal{Z}_t \right) C_t - A_t \right]$$
(44)

This implies that (43) can be rewritten as (42).

Finally, for aggregate durables and cash holdings we have

$$D_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} d_{z_{t-j},t} = \xi_t C_t, \qquad (45)$$

$$M_t \equiv \sum_{j=0}^{\infty} \varphi^j z_{t-j} m_{z_{t-j},t} = \iota_t C_t.$$
(46)

2.2 Aggregate asset stock

Financial wealth can be held as government bonds, foreign bonds, and claims to physical capital. Hence,

$$A_t = B_t + F_t + V_t, \tag{47}$$

where B_t represents the value of the end-of-period stock of government bonds, F_t the value of the end-ofperiod stock of foreign assets, and V_t the value of the end-of-period stock of claims to physical capital, all measured in consumption units. By assuming assets to be perfect substitutes in the household's portfolio, they earn the same (exogenous) real rate of return in equilibrium. We will now analyze in detail the different types of assets.

2.2.1 Physical capital accumulation

We are particularly interested in the cash flow from physical capital since, as mentioned, we assume that households as a whole, which can be considered as an investment firm, are also in charge of investment. More specifically, investment is determined by maximizing the cash flow from investing in physical capital, conditional on the law of motion of physical capital.

The cash flow from investing in physical capital is given by:

$$\sum_{s=t}^{\infty} \tilde{\mathbf{R}}_{t,s} \left\{ \left[(1 - \tau_K) \frac{r_s}{p_s} + \tau_K \delta_K \right] K_{s-1} - I_s \right\},\tag{48}$$

where $\tilde{\mathbf{R}}_{t,s} \equiv \prod_{j=t+1}^{s} [R_j (p_{j-1}/p_j)]^{-1}$ is the aggregate discount factor,³ r_t is the rental rate on capital, and I_t denotes investment. Note that the investment firm can deduct all depreciation from its taxable income. Physical capital evolves according to:

$$K_{t} = (1 - \delta_{K}) K_{t-1} + \Xi \left(\frac{I_{t}}{K_{t-1}}\right) K_{t-1},$$
(49)

where δ_K is the depreciation rate of capital and the term $\Xi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}$ indicates that there are adjustment costs. In particular, following Jermann (1998), we assume that those are

$$\Xi\left(\frac{I_t}{K_{t-1}}\right) = \frac{\Xi_1}{\varsigma} \left(\frac{I_t}{K_{t-1}}\right)^{\varsigma} + \Xi_2.$$
(50)

The two parameters Ξ_1 and Ξ_2 are designed to make the adjustment cost vanish in the steady state.

The Lagrangian is given by

$$\tilde{L}_{t} = \sum_{s=t}^{\infty} \tilde{R}_{t,s} \left\{ \left[(1 - \tau_{K}) \frac{r_{s}}{p_{s}} + \tau_{K} \delta \right] K_{s-1} - I_{s} + \nu_{s} \left[(1 - \delta_{K}) K_{s-1} + \Xi \left(\frac{I_{s}}{K_{s-1}} \right) K_{s-1} - K_{s} \right] \right\}.$$
(51)

The first order conditions (w.r.t. I_t and K_t) are:

$$\nu_t = \Xi' \left(\frac{I_t}{K_{t-1}} \right)^{-1}, \tag{52}$$

$$\nu_t = \frac{(1 - \tau_K) \frac{r_{t+1}}{p_{t+1}} + \tau_K \delta_K - \frac{I_{t+1}}{K_t} + \nu_{t+1} \left[1 - \delta_K + \Xi \left(\frac{I_{t+1}}{K_t} \right) \right]}{R_{t+1} \frac{p_t}{p_{t+1}}},$$
(53)

with the transversality condition (TVC):

$$\lim_{j \to \infty} \tilde{\mathbf{R}}_{t,j} \nu_j K_j = 0.$$
(54)

Equation (53) can be rewritten as the standard no-arbitrage condition:

$$R_{t+1} = \frac{(1 - \tau_K) r_{t+1} + p_{t+1} \left(\tau_K \delta_K - \frac{I_{t+1}}{K_t} \right) + p_{t+1} \nu_{t+1} \left[1 - \delta_k + \Xi \left(\frac{I_{t+1}}{K_t} \right) \right]}{p_t \nu_t},$$
(55)

where the last term on the right-hand side represents the future marginal contribution of capital to lower installation costs. In other words, the future net-of-tax gross return on claims to physical capital has to be

³Using (12), we can show that $\tilde{\mathbf{R}}_{t,s} \equiv \prod_{j=1}^{s} \left[R_{t+j} \frac{p_{t+j-1}}{p_{t+j}} \right]^{-1} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} \frac{p_t}{p_{t+s}}$, where λ_t is the aggregate shadow value of firms' profits in the household budget constraint.

equal to the future return of holding a unit of capital for one period (i.e. the future rental rate plus the future shadow price corrected for depreciation plus the future decrease in installation costs) divided by the current shadow price of the same unit of capital. Thus ν_t corresponds to the well-known Tobin q.

It can be easily shown that:

$$\nu_t K_t = \frac{\left[(1 - \tau_K) \frac{r_{t+1}}{p_{t+1}} + \tau_K \delta_K \right] K_t - I_{t+1} + \nu_{t+1} K_{t+1}}{R_{t+1} \frac{p_t}{p_{t+1}}}.$$
(56)

Hence, iterating on the previous expression and imposing the TVC yields:

$$\nu_t K_t = \sum_{s=t+1}^{\infty} \tilde{\mathbf{R}}_{t,s} \left\{ \left[(1 - \tau_K) \, r_s + \tau_K \delta_K p_s \right] K_{s-1} - p_s I_s \right\}.$$
(57)

The right-hand side in (57) represents the discounted flow of future cash flows in real terms, i.e. the stock market value of claims to physical capital. This implies that:

$$V_t = p_t \nu_t K_t. \tag{58}$$

2.2.2 Net foreign asset position

Combining (47), (35), (49) and (202), we get the following law of motion for net foreign assets:

$$F_{t} = R_{t}F_{t-1} + \mathcal{H}_{t} + \left[(1 - \tau_{K})r_{t} + \tau_{K}\delta_{K}p_{t} \right]K_{t-1} - \mathcal{Z}_{t}C_{t} - p_{t}I_{t} - (G_{t} - T_{t}).$$
(59)

2.3 Firms and Unions

Firms produce intermediate and final goods, and distribute imported intermediate goods. We assume that there is a single representative firm producing the final good Y under perfect competition. This firm combines \mathcal{N} intermediate goods using a CES production function, possibly with increasing returns in the variety of intermediate inputs.

Local firms in the intermediate goods sector produce N varieties of differentiated goods, operating under monopolistic competition. A share Θ of these N locally produced varieties cannot be traded (exported). The remaining $(1 - \Theta)$ can be exported.

Furthermore, other $(1 - \Theta^*) N^*$ varieties are imported and distributed, where N^{*} indicates the total number of foreign produced varieties, and Θ^* the share of them that can be imported in Luxembourg. Hence, the total number of varieties in Luxembourg is given by $\mathcal{N} = N + (1 - \Theta^*) N^*$. These foreign varieties are combined with capital and labour inputs by the distributors. The distributed intermediate goods can be either sold locally (to the final good producer) or exported. The first possibility represents the use of distributed imported varieties in the local economy while the second possibility captures re-export in the model, which helps to capture the particulalry large trade flows between Luxembourg and the rest of the world. The use of capital and labour in the distribution of these imported intermediate goods follows Mazenga and Ravn (1998, 2004).

Each firm in the intermediate goods sector adopts a nested CES production production function with capital and two different types of labour as inputs. The different types of labour are introduced to capture the dual labour market in Luxembourg, and represent resident and non-resident workers. The firm chooses the optimal demand of capital and each type of labour by maximizing profits subject to the production function constraint, taking wages and the cost of capital as given. The cost of capital is determined endogenously in order to match demand and supply of capital. For the sake of exposition, we will first present all the derivations for a generic production function, and then specialize the results to the nested CES case, which requires a more complex notation.

Each distributor of imported intermediate goods adopts a Leontief type production function, with imported foreign intermediate varieties and a nested CES combination of capital and two different types of labour as inputs. The firm chooses the optimal demand of the intermediate varieties imported, of capital and each type of labour by maximizing profits subject to the production function constraint, taking prices of the varieties, wages and the cost of capital as given.

Wages are determined by the interaction between the firms in the intermediate goods and distribution sector and the unions, which represent the workers (the so-called "right to manage" model). In particular, we assume that there is a union for each type of workers, and that bargaining with the firm takes place in a Nash setting. We assume that there is a separate union for each firm, but this is not a restrictive hypothesis since in symmetric equilibrium firms will make the same choices in terms of demand for labour and capital. Given the resulting wages, labour demand is determined, and it is assumed that for the current wages the supply of each type of labour adjusts to meet demand.

Technically, the interaction between the production and labour markets is represented as a game in two stages, where wage bargaining takes place in the first stage and production in the second. As in Lockwood (1990), the second stage is solved first, and the solution is used in the first stage. Therefore, after discussing the final good sector, we will first describe the problem of the firms (second stage), and then the firm-union bargain (first stage). We will deal, in turn, with producers of non-tradable goods, tradable goods, and importers of foreign intermediate goods.

2.3.1 Final good sector

The cost function for the final good producing firm is:

$$\mathcal{C}_F\left(\left\{p_j\right\}, Y\right) \equiv \min_{\left\{y_j\right\}} \sum_{j=1}^{\mathcal{N}} p_j y_j, \tag{60}$$

s.t.
$$\mathcal{N}^{\rho-\mu}\left(\sum_{j=1}^{\mathcal{N}} y_j^{\frac{1}{\mu}}\right)^{\mu} \ge Y,$$
 (61)

where y_j is the amount of the j^{th} intermediate good used for production of the final good $Y, j = 1, ..., \mathcal{N}$ (where \mathcal{N} is the total number of domestic and imported varieties); $\mu > 1$ is indirectly related to the elasticity of substitution between goods and directly related to the mark-up in the intermediate goods sector; and $\rho \geq 1$ is a parameter that captures increasing returns to variety; see Kim (2004) for details.

Writing the Lagrangean function as:

$$\mathcal{L} = \sum_{j=1}^{\mathcal{N}} p_j y_j + \lambda \left[Y - \mathcal{N}^{\rho-\mu} \left(\sum_{j=1}^{\mathcal{N}} y_j^{\frac{1}{\mu}} \right)^{\mu} \right], \tag{62}$$

the first order conditions are:

$$y_j = \mathcal{N}^{\frac{\rho-\mu}{\mu-1}} \left(\frac{p_j}{\lambda}\right)^{\frac{\mu}{1-\mu}} Y.$$
(63)

Hence, for any $s, j \in \{0, 1, ..., \mathcal{N}\}$:

$$y_s = \left(\frac{p_s}{p_j}\right)^{\frac{\mu}{1-\mu}} y_j. \tag{64}$$

Substituting (64) into (61) and simplifying, we get the conditional demand for intermediate good j:

$$y_{j} = \frac{p_{j}^{\frac{1}{p-\mu}}Y}{\mathcal{N}^{\rho-\mu} \left(\sum_{s=1}^{\mathcal{N}} p_{s}^{\frac{1}{1-\mu}}\right)^{\mu}}.$$
(65)

We can then write the unit cost function as:

$$C_F(\{p_j\}, 1) = p = \mathcal{N}^{-(\rho-\mu)} \left(\sum_{j=1}^{\mathcal{N}} p_j^{\frac{1}{1-\mu}}\right)^{1-\mu},$$
(66)

and, therefore, express the conditional demand for intermediate good j as

$$y_j = \left(\frac{p_j}{p}\right)^{\frac{\mu}{1-\mu}} Y \mathcal{N}^{\frac{\rho-\mu}{\mu-1}},\tag{67}$$

or:

$$p_j = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{y_j}{Y}\right)^{\frac{1-\mu}{\mu}} p. \tag{68}$$

2.3.2 Intermediate goods sector - Non-tradable goods: $j \in [1, \Theta N]$

Second stage: profit maximization The problem of a generic firm in the intermediate goods sector producing a non-tradable good can be formulated as

$$\max_{\{h_{zj}^{NT}, k_{j}^{NT}\}} \pi_{j}^{NT} \equiv p_{j}^{NT} (y_{j}^{NT}) y_{j}^{NT} - (1 + \psi_{K} i^{W}) r k_{j}^{NT} + (1 + \tilde{\tau}_{L}) (1 + \psi_{L} i^{W}) \sum_{z=1}^{2} w_{zj}^{NT} h_{zj}^{NT} - \psi_{j},$$
(69)

where $p(y_j^{NT})$ indicates the price of the j^{th} non-tradable intermediate good; h_{zj}^{NT} , z = 1, 2, the amount of the two types of labour (resident and non-resident) and and k_j^{NT} capital; ψ_j is a fixed financial cost to enter the market (the fixed cost generates economies of scale and therefore justifies monopolistic competition; see Kim, 2004); ψ_L and ψ_K denote the fraction of wage and rental bills, respectively, that must be financed in advance while i_t^W denotes the net interest rate firms pay on the working-capital to meet these advance payments⁴;

⁴Notice that we assume firms pay taxes on labour, $\tilde{\tau}_L w_{zj}^{NT} h_{zj}^{NT}$, at the same time when wages. Thus, firms must also borrow working capital to pay ψ_L fraction of the labour tax in advance. Another option would be to have wage costs entering the firms' profit function formulated as $(1 + \tilde{\tau}_L + \psi_L i_t^W) w_{zj}^{NT} h_{zj}^{NT}$. The underlying assumption behind this formulation is that taxes are paid at the end of the period and thus working capital is required only for the advance payment of net wages. Since in our model a period is one year and firms pay taxes more frequently than annually we choose the first formulation.

and $\tilde{\tau}_L$ represents taxes on labour (social contributions) paid by firms; labour income taxes paid by workers will be taken into account later. In addition:

$$p_j^{NT}\left(y_j^{NT}\right) = \mathcal{N}^{\frac{\rho-\mu}{\mu}}\left(\frac{y_j^{NT}}{Y}\right)^{\frac{1-\mu}{\mu}}p,\tag{70}$$

$$y_j^{NT} = f\left(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT}\right),$$
(71)

where the specific functional form for the production function will be discussed later on.

The first order conditions are:

$$\left(\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT}\right) \frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}} = (1 + \tilde{\tau}_L) \left(1 + \psi_L i^W\right) w_{zj}^{NT}, \tag{72}$$

$$\left(\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT}\right) \frac{\partial y_j^{NT}}{\partial k_j^{NT}} = \left(1 + \psi_K i^W\right) r,\tag{73}$$

where $z \in \{1, 2\}$.

Note that (if the firm takes P and Y as given):

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} y_j^{NT} + p_j^{NT} = \frac{p_j^{NT}}{\mu}.$$
(74)

Hence:

$$p_j^{NT} \frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}} = \mu \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right) w_{zj}^{NT}.$$

$$\tag{75}$$

$$p_j^{NT} \frac{\partial y_j^{NT}}{\partial k_j^{NT}} = \mu \left(1 + \psi_K i^W \right) r.$$
(76)

Conditionally on k_j^{NT} and, respectively, n_{2j}^{NT} and n_{1j}^{NT} , (75) implicitly defines the conditional demands for the two types of labour:

$$h_{zj}^{NT} = h_{zj}^{NT} \left(w_{zj}^{NT} \right).$$
(77)

Note that, thanks to the Envelope Theorem, (72) implies:

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}}\right)^2 \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} + p_j^{NT} \frac{\partial^2 y_j^{NT}}{(\partial h_{zj}^{NT})^2} \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} = \mu \left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right). \tag{78}$$

Hence:

$$\frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} = \frac{\mu \left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right)}{\frac{1 - \mu}{\mu} \frac{p_j^{NT}}{y_j^{NT}} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}}\right)^2 + p_j^{NT} \frac{\partial^2 y_j^{NT}}{(\partial h_{zj}^{NT})^2}}{(\partial h_{zj}^{NT})^2} = \frac{1}{\frac{1}{w_{zj}^{NT}}} \left[\left(1 - \mu\right) \frac{\left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right) w_{zj}^{NT}}{p_j^{NT} y_j^{NT}} + \frac{\partial^2 y_j^{NT}}{(\partial h_{zj}^{NT})^2} \left(\frac{\partial y_j^{NT}}{\partial h_{zj}^{NT}}\right)^{-1} \right]^{-1}, \quad (79)$$

since:

$$\frac{\partial p_j^{NT}}{\partial y_j^{NT}} = \frac{1-\mu}{\mu} \frac{p_j^{NT}}{y_j^{NT}}.$$
(80)

First stage: firm-union bargaining (Labour market) We follow the standard right-to-manage approach and assume that each firm-union pair bargains over type-z wage, taking the labour demand curve into account. The outcome of the bargaining process can be depicted as the solution of the following maximization problem:

$$\max_{w_{z_j}^{NT}} \Omega^{NT} \equiv \left[(1 - \tau_L) \left(\frac{w_{z_j}^{NT}}{p} - \frac{\bar{w}_z}{p} \right) h_{z_j}^{NT} \right]^{\theta_z^{NT}} \left[\frac{\tilde{\pi} \left(w_{z_j}^{NT} \right)}{p} \right]^{1 - \theta_z^{NT}}, \tag{81}$$

where θ_z^{NT} is a parameter describing the relative bargaining power of the union for type z workers (possibly sector-specific), \bar{w}_z the workers' outside option, and:

$$\tilde{\pi}^{NT} \left(w_{zj}^{NT} \right) = p^{NT} \left[f \left(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT} \right) \right] f \left(k_j^{NT}, h_{1j}^{NT}, h_{2j}^{NT} \right) +$$
(82)

$$-(1+\tilde{\tau}_L)\left(1+\psi_L i^W\right)\sum_{z=1}^2 w_{zj}^{NT} h_{zj}^{NT}.$$
(83)

Hence, the unions care about a convex combination of the total wage bill and profits.

For z = 1, 2, the first order conditions read as:

$$\theta_{z}^{NT}\tilde{\pi}_{j}^{NT}\left[h_{zj}^{NT} + \left(w_{zj}^{NT} - \bar{w}_{z}\right)\frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}}\right] + \left(1 - \theta_{z}^{NT}\right)\left(w_{zj}^{NT} - \overline{w}_{z}\right)h_{zj}^{NT}\frac{\partial \tilde{\pi}_{j}^{NT}}{\partial w_{zj}^{NT}} = 0, \quad (84)$$

where:

$$\frac{\partial \tilde{\pi}_{j}^{NT}}{\partial w_{zj}^{NT}} = \underbrace{\left(\frac{\partial p_{j}^{NT}}{\partial y_{j}^{NT}} y_{j}^{NT} + p_{j}^{NT}\right) \frac{\partial y_{j}^{NT}}{\partial h_{zj}^{NT}}}_{(1+\tilde{\tau}_{L})\left(1+\psi_{L}i_{t}^{W}\right)w_{zj}} \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} + \left(1+\tilde{\tau}_{L}\right)\left(1+\psi_{L}i_{t}^{W}\right)w_{zj}} - \left(1+\tilde{\tau}_{L}\right)\left(1+\psi_{L}i^{W}\right)\left(h_{zj}^{NT} + w_{zj}^{NT}\frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}}\right) = -\left(1+\tilde{\tau}_{L}\right)\left(1+\psi_{L}i^{W}\right)h_{zj}^{NT}. \quad (85)$$

Therefore,

$$\theta_z^{NT} \left(1 + \frac{w_{zj}^{NT} - \overline{w}_{z,t}}{w_{zj}^{NT}} \epsilon_{zj}^{NT} \right) \frac{\tilde{\pi}_j^{NT}}{h_{zj}^{NT}} = \left(1 - \theta_z^{NT} \right) \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right) \left(w_{zj}^{NT} - \bar{w}_{z,t} \right) \tag{86}$$

where:

$$\epsilon_{zj} \equiv \frac{\partial h_{zj}^{NT}}{\partial w_{zj}^{NT}} \frac{w_{zj}^{NT}}{h_{zj}^{NT}}$$
(87)

We will derive similar equations for the tradable intermediate goods sector in the next subsection. Several factors affect real wages in LSM2. First, as usual, labour productivity. Second, the characteristics of the labour market, such as the union power θ and the replacement ratios \bar{w}_j/w_j . Third, the profit rate, since unions extract some of the producer surplus. Fourth, the relative productivity of the two types of labour, the relative size of the labour forces, and the unemployment rates. Finally, the relative productivity with respect to capital and the amount of capital per worker.

2.3.3 Intermediate goods sector - Tradable goods: $j \in [\Theta N, N]$

Second stage: profit maximization Let us now consider the problem of a generic firm in the intermediate goods sector producing tradable goods, y_j^T , such that $y_j^H = s_j^H y_j^T$ is sold at home and $y_j^F = s_j^F y_j^T$ is exported $(s_j^F = 1 - s_j^H)$, and $0 \le s_j^H \le 1$, with corresponding prices given by p_j^H and p_j^F . The firm should choose the amount of labour and capital to be used for the production of y_j^T ($h_{z_j}^T$ and k_j^T , respectively, z = 1, 2), and the share of y_j^T sold at home, s_j^H , to optimize the following problem:

$$\max_{\left\{h_{z_{j}}^{T},k_{j}^{T},s_{j}^{H}\right\}} \pi_{j}^{T} \equiv p_{j}^{T}\left(y_{j}^{T}\right)y_{j}^{T} - \left(1 + \psi_{K}i^{W}\right)rk_{j}^{T} - \left(1 + \tilde{\tau}_{L}\right)\left(1 + \psi_{L}i^{W}\right)\sum_{z=1}^{2}w_{z_{j}}^{T}h_{z_{j}}^{T} - \psi_{j},$$

where:

$$p_{j}^{T} = s_{j}^{H} p_{j}^{H} + s_{j}^{F} p_{j}^{F}, (88)$$

$$s_j^F = 1 - s_j^H, (89)$$

$$y_j^T = f\left(k_j^T, h_{1j}^T, h_{2j}^T\right),$$
(90)

$$p_j^H = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s_j^H y_j^T}{Y}\right)^{\frac{\mu}{\mu}} p, \qquad (91)$$

$$p_{j}^{F} = (1 - t^{F}) \left(\mathcal{N}^{*} \right)^{\frac{\rho - \mu}{\mu}} \left(\frac{s_{j}^{F} y_{j}^{T}}{Y^{*}} \right)^{\frac{1 - \mu}{\mu}} p^{*}.$$
(92)

Note that Y^* and p^* represent foreign output and the foreign aggregate price. Furthermore, note that the elasticity of substitution between intermediate goods is the same at home and abroad, i.e. $\mu^* = \mu$: this assumption is maintained for notational simplicity, but the model can be easily generalized.⁵ As in the non-tradable sector, ψ_j is a fixed financial cost to enter the market that generates economies of scale and therefore provides a basis for monopolistic competition; see Kim, 2004.

The first order conditions are:

$$\left(\frac{\partial p_j^T}{\partial y_j^T}y_j^T + p_j^T\right)\frac{\partial y_j^T}{\partial h_{zj}^T} = (1 + \tilde{\tau}_L)\left(1 + \psi_L i^W\right)w_{zj}^T,\tag{93}$$

$$\left(\frac{\partial p_j^T}{\partial y_j^T} y_j^T + p_j^T\right) \frac{\partial y_j^T}{\partial k_j^T} = \left(1 + \psi_K i^W\right) r,\tag{94}$$

$$p_j^H = p_j^F. (95)$$

where $z \in \{1, 2\}$.

Assuming if the firm takes P and Y as given:

$$\frac{\partial p_j^T}{\partial y_j^T} = \frac{1-\mu}{\mu} \frac{s_j^H p_j^H + s_j^F p_j^F}{y_j^T} = \frac{1-\mu}{\mu} \frac{p_j^T}{y_j^T}.$$
(96)

Hence:

$$\frac{\partial p_j^T}{\partial y_j^T} y_j^T + p_j^T = \frac{p_j^T}{\mu}.$$
(97)

 $^{{}^{5}}$ The distinction between local and foreign elasticities is important to study shocks to local markups that do not transmit to markups in foreign markets. In this case, we obviously use the generalized version of the model.

This implies that:

$$p_j^T \frac{\partial y_j^T}{\partial h_{z_j}^T} = \mu \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right) w_{z_j}^T, \tag{98}$$

$$p_j^T \frac{\partial y_j^T}{\partial k_j^T} = \mu \left(1 + \psi_K i^W \right) r.$$
⁽⁹⁹⁾

Conditionally on k_j^T and, respectively, h_{2j}^T and h_{1j}^T , the FOCs implicitly define the conditional demands for the two types of labour:

$$h_{zj}^{T} = h_{zj}^{T} \left(w_{zj}^{T} \right).$$
 (100)

Finally, from the Envelope Theorem:

$$\frac{\partial p_j^T}{\partial y_j^T} \left(\frac{\partial y_j^T}{\partial h_{zj}^T} \right)^2 \frac{\partial h_{zj}^T}{\partial w_{zj}^T} + \tilde{p}_j^T \frac{\partial^2 y_j^T}{\left(\partial h_{zj}^T\right)^2} \frac{\partial h_{zj}^T}{\partial w_{zj}^T} = \mu \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right). \tag{101}$$

Hence:

$$\frac{\partial h_{zj}^{T}}{\partial w_{zj}^{T}} = \frac{\mu \left(1 + \tilde{\tau}_{L}\right) \left(1 + \psi_{L} i_{t}^{W}\right)}{\frac{1 - \mu}{\mu} \frac{p_{j}^{T}}{y_{j}^{T}} \left(\frac{\partial y_{j}^{T}}{\partial h_{zj}^{T}}\right)^{2} + p_{j}^{T} \frac{\partial^{2} y_{j}^{T}}{\left(\partial h_{zj}^{T}\right)^{2}}}{\frac{1}{w_{zj}^{T}}} = \frac{1}{\frac{1}{w_{zj}^{T}}} \left[\left(1 - \mu\right) \frac{\left(1 + \tilde{\tau}_{L}\right) \left(1 + \psi_{L} i^{W}\right) w_{zj}^{T}}{p_{j}^{T} y_{j}^{T}} + \frac{\partial^{2} y_{j}^{T}}{\left(\partial h_{zj}^{T}\right)^{2}} \left(\frac{\partial y_{j}^{T}}{\partial h_{zj}^{T}}\right)^{-1} \right]^{-1}. \quad (102)$$

First stage: firm-union bargaining (Labour market) The firm-union bargain is similar to that in the non-tradable sector. In particular, the counterpart of equation (81) is:

$$\max_{w_{zj}^T} \Omega \equiv \left[(1 - \tau_L) \left(\frac{w_{zj}^T}{P} - \frac{\overline{w}_z}{P} \right) h_{zj}^T \right]^{\theta_z^T} \left[\frac{\tilde{\pi}^T \left(w_{zj}^T \right)}{P} \right]^{1 - \theta_z^T},$$
(103)

where:

$$\tilde{\pi}^{T}\left(w_{zj}^{T}\right) = p_{j}^{T}\left[f\left(h_{1j}^{T}, h_{2j}^{T}, k_{j}^{T}\right)\right]f\left(h_{1j}^{T}, h_{2j}^{T}, k_{j}^{T}\right) - (1 + \tilde{\tau}_{L})\left(1 + \psi_{L}i^{W}\right)\sum_{s=1}^{2}w_{sj}^{T}h_{sj}^{T}.$$
(104)

The first order conditions are:

$$\theta_z^T \tilde{\pi}_j^T \left[h_{zj}^T + \left(w_{zj}^T - \overline{w}_{z,t} \right) \frac{\partial h_{zj}^T}{\partial w_{zj}^T} \right] + \left(1 - \theta_z^T \right) \left(w_{zj}^T - \overline{w}_{z,t} \right) h_{zj}^T \frac{\partial \tilde{\pi}_j^T}{\partial w_{zj}^T} = 0, \tag{105}$$

where:

$$\frac{\partial \tilde{\pi}_j^T}{\partial w_{zj}^T} = -\left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right) h_{zj}^T.$$
(106)

Hence:

$$\theta_z^T \left(1 + \frac{w_{zj}^T - \overline{w}_z}{w_{zj}^T} \epsilon_{zj}^T \right) \frac{\tilde{\pi}_j^T}{h_{zj}^T} = \left(1 - \theta_z^T \right) \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right) \left(w_{zj}^T - \overline{w}_z \right), \tag{107}$$

where:

$$\tilde{\pi}_{j}^{T} = p_{j}^{T} y_{j}^{T} - (1 + \tilde{\tau}_{L}) \left(1 + \psi_{L} i^{W} \right) \sum_{z=1}^{2} w_{zj}^{T} h_{zj}^{T}.$$
(108)

$$\epsilon_{zj}^{T} \equiv \frac{\partial h_{zj}^{T}}{\partial w_{zj}^{T}} \frac{w_{zj}^{T}}{h_{zj}^{T}}.$$
(109)

2.3.4 Intermediate goods sector - Distribution of imported varieties

Second stage: profit maximization The generic distributor of imported intermediate varieties produces tradable intermediate goods, y_j^M , such that $y_j^{M,H} = s_j^{M,H}y_j^M$ is sold in the home market and $y_j^{M,F} = (1 - s_j^{M,H})y_j^M$ is re-exported ($0 \le s_j^{M,H} \le 1$). The corresponding prices are $p_j^{M,H}$ and $p_j^{M,F}$. The main feature that distinguishes the distributor of imported varieties from the tradable intermediate goods producers is that the former use foreign varieties imported at the price p_M^* as well as capital and labour to produce their services of distribution. Apart from this difference the profit maximization problems in the two sectors are very similar:

$$\max_{\left\{x_{j}^{M},h_{zj}^{M},k_{j}^{M},s_{j}^{M,H}\right\}} \pi_{j}^{M} \equiv p_{j}^{M}\left(y_{j}^{M}\right)y_{j}^{M} - \left(1 + t^{M}\right)p_{M}^{*}x_{j}^{M} + (110)
- \left(1 + \psi_{K}i^{W}\right)rk_{j}^{M} - \left(1 + \tilde{\tau}_{L}\right)\left(1 + \psi_{L}i^{W}\right)\sum_{z=1}^{2}w_{zj}^{M}h_{zj}^{M} - \psi_{j},$$

where x_j^M denotes the imported amount of the foreign intermediate good. The maximization problem is subject to:

$$p_j^M = s_j^{M,H} p_j^{M,H} + s_j^{M,F} p_j^{M,F}, \qquad (111)$$

$$s_j^{M,F} = 1 - s_j^{M,H},$$
 (112)

$$y_{j}^{M} = \min\left[\pi_{1}x_{j}^{M}, f^{M}\left(k_{j}^{M}, h_{j}^{M}\right)\right], \qquad (113)$$

$$p_j^{M,H} = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s_j^{M,H} y_j^M}{Y}\right)^{\frac{-\mu}{\mu}} p, \qquad (114)$$

$$p_{j}^{M,F} = \left(1 - t^{F}\right) \left(\mathcal{N}^{*}\right)^{\frac{\rho - \mu}{\mu}} \left(\frac{s_{j}^{M,F} y_{j}^{M}}{Y^{*}}\right)^{\frac{1 - \mu}{\mu}} p^{*}.$$
(115)

Equation (111) implies that imports of foreign intermediate goods are perfect complements to local distribution services, produced via capital and labor, as in Ravn and Mazzenga (2004). Cost minimization implies that $\pi_1 x_j^M = f^M (k_j^M, h_j^M)$; therefore, in equilibrium $x_j^M = y_j^M / \pi_1$. For the sake of simplicity, let's anticipate this equilibrium outcome, and rewrite the maximization problem as:

$$\max_{\left\{h_{zj}^{M},k_{j}^{M},s_{j}^{M,H}\right\}} \pi_{j}^{M} \equiv \left[p_{j}^{M}\left(y_{j}^{M}\right) - \left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}\right]y_{j}^{M} + \left(116\right) - \left(1 + \psi_{K}i^{W}\right)rk_{j}^{M} - \left(1 + \tilde{\tau}_{L}\right)\left(1 + \psi_{L}i^{W}\right)\sum_{z=1}^{2}w_{zj}^{M}h_{zj}^{M} - \psi_{j}.$$

The first order conditions are then given by:

$$\left[\frac{\partial p_j^M}{\partial y_j^M}y_j^M + p_j^M - \left(1 + t^M\right)\frac{p_M^*}{\pi_1}\right]\frac{\partial y_j^M}{\partial h_{zj}^M} = (1 + \tilde{\tau}_L)\left(1 + \psi_L i_t^W\right)w_{zj}^M,\tag{117}$$

$$\left[\frac{\partial p_j^M}{\partial y_j^M}y_j^M + p_j^M - \left(1 + t^M\right)\frac{p_M^*}{\pi_1}\right]\frac{\partial y_j^M}{\partial k_j^M} = \left(1 + \psi_K i_t^W\right)r,\tag{118}$$

$$p_j^{M,H} = p_j^{M,F}.$$
 (119)

where $z \in \{1, 2\}$.

If the firm takes P and Y as given:

$$\frac{\partial p_j^M}{\partial y_j^M} = \frac{1-\mu}{\mu} \frac{s_j^{M,H} p_j^{M,H} + s_j^{M,F} p_j^{M,F}}{y_j^M} = \frac{1-\mu}{\mu} \frac{p_j^M}{y_j^M}.$$
(120)

Hence:

$$\frac{\partial p_j^M}{\partial y_j^M} y_j^M + p_j^M = \frac{p_j^M}{\mu}.$$
(121)

This implies that:

$$\left[p_j^M - \mu \left(1 + t^M\right) \frac{p_M^*}{\pi_1}\right] \frac{\partial y_j^M}{\partial h_{zj}^M} = \mu \left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right) w_{zj}^M, \tag{122}$$

$$\left[p_j^M - \mu \left(1 + t^M\right) \frac{p_M^*}{\pi_1}\right] \frac{\partial y_j^M}{\partial k_j^M} = \mu \left(1 + \psi_K i^W\right) r.$$
(123)

The resulting profits are:

$$\pi_j^M = \left(1 - \frac{1}{\mu}\right) p_j^M y_j^M - \psi_j$$

while the value added amounts to:

$$va_{j}^{M} = \left[p_{j}^{M} - \left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}\right]y_{j}^{M} - \psi_{j}.$$
(124)

Conditionally on k_j^M , x_j^M and, respectively, h_{2j}^M and h_{1j}^M , the FOCs implicitly define the conditional demands for the two types of labour:

$$h_{zj}^M = h_{zj}^M \left(w_{zj}^M \right). \tag{125}$$

Finally, from the Envelope Theorem:

$$\left\{ \frac{\partial p_j^M}{\partial y_j^M} \left(\frac{\partial y_j^M}{\partial h_{zj}^M} \right)^2 + \left[p_j^M - \mu \left(1 + t^M \right) \frac{p_M^*}{\pi_1} \right] \frac{\partial^2 y_j^M}{\left(\partial h_{zj}^M \right)^2} \right\} \frac{\partial h_{zj}^M}{\partial w_{zj}^M} = \mu \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right). \tag{126}$$

Hence:

$$\frac{\partial h_{zj}^{M}}{\partial w_{zj}^{M}} = \frac{1}{w_{zj}^{M}} \frac{\left[p_{j}^{M} - \mu \left(1 + t^{M} \right) \frac{p_{M}^{*}}{\pi_{1}} \right] \frac{\partial y_{j}^{M}}{\partial h_{zj}^{M}}}{\frac{1 - \mu}{\mu} \frac{p_{j}^{M}}{y_{j}^{M}} \left(\frac{\partial y_{j}^{M}}{\partial h_{zj}^{M}} \right)^{2} + \left[p_{j}^{M} - \mu \left(1 + t^{M} \right) \frac{p_{M}^{*}}{\pi_{1}} \right] \frac{\partial^{2} y_{j}^{M}}{\left(\partial h_{zj}^{M} \right)^{2}} = \frac{1}{w_{zj}^{M}} \left[\frac{\frac{1 - \mu}{\mu} \frac{p_{j}^{M}}{y_{j}^{M}} \frac{\partial y_{j}^{M}}{\partial h_{zj}^{M}}}{p_{j}^{M} - \mu \left(1 + t^{M} \right) \frac{p_{M}^{*}}{\pi_{1}}} + \frac{\partial^{2} y_{j}^{M}}{\left(\partial h_{zj}^{M} \right)^{2}} \left(\frac{\partial y_{j}^{M}}{\partial h_{zj}^{M}} \right)^{-1} \right]^{-1}.$$
 (127)

First stage: firm-union bargaining (Labour market) The firm-union bargain is similar to that in the non-tradable sector. In particular, the counterpart of equation (81) is:

$$\max_{w_{zj}^T} \Omega \equiv \left[(1 - \tau_L) \left(\frac{w_{zj}^M}{P} - \frac{\bar{w}_{z,t}}{P} \right) h_{zj}^M \right]^{\theta_z^M} \left[\frac{\tilde{\pi}^M \left(w_{zj}^M \right)}{P} \right]^{1 - \theta_z^M}.$$
(128)

where:

$$\tilde{\pi}^{M}\left(w_{zj}^{M}\right) = \left[p_{j}^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}\right]y_{j}^{M} - (1 + \tilde{\tau}_{L})\left(1 + \psi_{L}i^{W}\right)\sum_{s=1}^{2}w_{sj}^{M}h_{sj}^{M}.$$
(129)

The first order conditions are

$$\theta_z^M \tilde{\pi}_j^M \left[h_{zj}^M + \left(w_{zj}^M - \bar{w}_{z,t} \right) \frac{\partial h_{zj}^M}{\partial w_{zj}^M} \right] + \left(1 - \theta_z^M \right) \left(w_{zj}^M - \bar{w}_{z,t} \right) h_{zj}^M \frac{\partial \tilde{\pi}_j^M}{\partial w_{zj}^M} = 0, \tag{130}$$

where:

$$\frac{\partial \tilde{\pi}_j^M}{\partial w_{zj}^M} = -\left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right) h_{zj}^M. \tag{131}$$

Hence:

$$\theta_z^M \left(1 + \frac{w_{zj}^M - \bar{w}_{z,t}}{w_{zj}^M} \epsilon_{zj}^M \right) \frac{\tilde{\pi}_j^M}{h_{zj}^M} = (1 + \tilde{\tau}_L) \left(1 + \psi_L i^W \right) \left(1 - \theta_z^M \right) \left(w_{zj}^M - \bar{w}_z \right), \tag{132}$$

where:

$$\tilde{\pi}_{j}^{M} = \left[p_{j}^{M} - \mu \left(1 + t^{M} \right) \frac{p_{M}^{*}}{\pi_{1}} \right] y_{j}^{M} - \left(1 + \tilde{\tau}_{L} \right) \left(1 + \psi_{L} i^{W} \right) \sum_{z=1}^{2} w_{zj}^{M} h_{zj}^{M}, \tag{133}$$

$$\epsilon_{zj}^{M} \equiv \frac{\partial h_{zj}^{M}}{\partial w_{zj}^{M}} \frac{w_{zj}^{M}}{h_{zj}^{M}}.$$
(134)

2.4 Domestic banking sector

There exists a representative, competitive bank that intermediates loans between households and firms, producing financial services using capital, labor and reserves. The bank is owned by the households. As in Christiano and Eichenbaum (1995) and Christiano, Motto and Rostagno (2008), firms (and banks themselves) need to finance a given fraction of their factor expenditure in advance, before revenues from sales can be cashed in, i.e. firms need short-term working capital.

To finance working-capital loans, the bank issues demand deposit liabilities to households. Working capital loans are made in form of demand deposits to firms. Hence, total deposits are equal to:

$$D^{T} = \underbrace{D^{H}}_{\text{Dep. to households}} + \underbrace{D^{F}}_{\text{Dep. to firms}}.$$
(135)

In equilibrium, demand deposits to households will match the supply of money not held for transaction services:

$$D^H = \bar{M} - M = \bar{M} - \iota C, \tag{136}$$

where ι is the equilibrium value related to equation (22), while demand deposits to firms will be equal to the short term working capital:

$$D^{F} = \psi_{K} r K + \psi_{L} \sum_{z=1}^{2} w_{z} H_{z}.$$
(137)

The parameters ψ_L and ψ_K represent the fraction of wage and rental bills, respectively, that must be financed in advance. Demand deposits pay an interest rate i^D . Interest on demand deposits created when firms and banks receive their working-capital loans are paid to the recipient of the loan. Firms and banks hold these demand deposits until the factor bill is paid in a settlement period that occurs after the goods market. Interest paid by firms on working capital loans is denoted $i^W + i^D$. Since firms receive interest payments on deposits, the net interest rate is i_t^W . Working-capital loans and demand deposits share the same maturity. Loans are extended just prior to production, and then paid off after production. Households deposit funds into the bank just prior to production, and then liquidate the deposit after production.

Demand deposits are associated with financial services. The bank has a technology for converting labor and capital services, and excess reserves, into transaction services:

$$\frac{D^T}{p} = Az^{CB} \left[\left(K^{CB} \right)^{\alpha^{CB}} \left(\Lambda H^{CB} \right)^{1-\alpha^{CB}} \right]^{\xi^{CB}} \left(\frac{E^{CB}}{p} \right)^{1-\xi^{CB}}, \tag{138}$$

where E^{CB} denote excess reserves, and:

$$H^{CB} = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} H_{z}^{CB}\right)^{\kappa}\right]^{\frac{1}{\kappa}},$$
(139)

where $\varkappa_1 + \varkappa_2 = 1$. Note that:

$$\frac{\partial H^{CB}}{\partial H_z^{CB}} = a_z \varkappa_z \left(\frac{H^{CB}}{a_z H_z^{CB}}\right)^{1-\kappa}.$$
(140)

Recall that loans are not used until the end of the period; excess reserves are then defined as:

$$E^{CB} = D^H - \varrho^{CB} D^T, \tag{141}$$

where ρ^{CB} denotes the required reserves coefficient. Excess reserves enter the production function in order to capture the idea that banks held excess reserves for a precautionary motif, given the possibility of unexpected withdrawals.

After the goods market clears, the bank settles claims for transactions that occurred in this market. The bank's sources of funds are: interest and principal on working capital loans, plus the resources it received from households at the start of the period. The bank's uses of funds are: principal and interest payments on demand deposits, plus gross expenses on labor and capital services. Then, the bank's net source of funds at the end of the period is:

$$\Pi^{CB} = D^{H} + \left(1 + i^{D} + i^{W}\right) D^{F} - \left(1 + i^{D}\right) \left(D^{H} + D^{F}\right) + \left(1 + \psi_{K} i^{W}\right) r K^{CB} - \left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) \sum_{z=1}^{2} w_{z}^{CB} H_{z}^{CB}, \quad (142)$$

that can be compactly rewritten as:

$$\Pi^{CB} = i^{W} D^{F} - i^{D} D^{H} - \left(1 + \psi_{K} i^{W}\right) r K^{CB} - \left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) \sum_{z=1}^{2} w_{z}^{CB} H_{z}^{CB}.$$
(143)

The bank solves the following maximization problem:

$$\begin{split} \max_{\{D^{F}, D^{H}, K^{CB}, H_{z}^{CB}\}} & \Pi^{CB} = i^{W} D^{F} - i^{D} D^{H} - \left(1 + \psi_{K} i^{W}\right) r K^{CB} + \\ & - \left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) \sum_{z=1}^{2} w_{z}^{CB} H_{z}^{B}, \\ \text{s.t.} & \frac{D^{H} + D^{F}}{p} = A z^{CB} \left[\left(K^{CB}\right)^{\alpha^{CB}} \left(\Lambda H^{CB}\right)^{1 - \alpha^{CB}} \right]^{\xi^{CB}} \left(\frac{E^{CB}}{p}\right)^{1 - \xi^{CB}} \end{split}$$

where:

$$E^{CB} = \left(1 - \varrho^{CB}\right)D^H - \varrho^{CB}D^F.$$
(145)

2.4.1 Profit maximization

The previous problem can be rewritten as:

$$\max_{\{D^{F}, E^{r}, K^{CB}, H_{z}^{CB}\}} \Pi^{CB} = i^{W} D^{F} - i^{D} \frac{E^{CB} + \varrho^{CB} D^{F}}{1 - \varrho^{CB}} +$$
(146)
$$- \left(1 + \psi_{K} i^{W}\right) r K^{CB} - \left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) \sum_{z=1}^{2} w_{z}^{CB} H_{z}^{CB},$$

s.t.
$$\frac{E^{CB} + D^{F}}{(1 - \varrho^{CB}) p} = Az^{CB} \left[\left(K^{CB}\right)^{\alpha^{CB}} \left(\Lambda H^{CB}\right)^{1 - \alpha^{CB}} \right]^{\xi^{CB}} \left(\frac{E^{CB}}{p}\right)^{1 - \xi^{CB}}.$$
(147)

We can form a Lagrangian:

$$\mathcal{L} = i^{W} D^{F} - i^{D} \frac{E^{CB} + \varrho^{CB} D^{F}}{1 - \varrho^{CB}} + - \left(1 + \psi_{K} i^{W}\right) r K^{CB} - \left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) \sum_{z=1}^{2} w_{z}^{CB} H_{z}^{CB} + \frac{\lambda^{CB}}{p} \left\{ pAz^{CB} \left[\left(K^{CB}\right)^{\alpha^{CB}} \left(\Lambda H^{CB}\right)^{1 - \alpha^{CB}} \right]^{\xi^{CB}} \left(\frac{E^{CB}}{p}\right)^{1 - \xi^{CB}} - \frac{E^{CB} + D^{F}}{1 - \varrho^{CB}} \right\}.$$
(148)

and compute the FOCs (w.r.t. $D^F, E^{CB}, K^{CB}, H_z^{CB}, \lambda^{CB})$:

$$\left(1-\varrho^{CB}\right)\left(i^W+i^D\right)-i^D=\frac{\lambda^{CB}}{p},\tag{149}$$

$$i^{D} = \frac{\lambda^{CB}}{p} \left[\left(1 - \xi^{CB} \right) \left(1 - \varrho^{CB} \right) \frac{D^{T}}{E^{CB}} - 1 \right], \tag{150}$$

$$\left(1 + \psi_K i^W\right) r = \frac{\lambda^{CB}}{p} \xi^{CB} \alpha^{CB} \frac{D^T}{K^{CB}},\tag{151}$$

$$\left(1+\psi_L i^W\right)\left(1+\tilde{\tau}_L\right)w_z^{CB} = \frac{\lambda^{CB}}{p}\xi^{CB}\left(1-\alpha^{CB}\right)\varkappa_z \frac{D^T}{H_z^{CB}}\left(\frac{a_z H_z^{CB}}{H^{CB}}\right)^{\kappa},\tag{152}$$

$$\frac{D^{T}}{p} = Az^{CB} \left[\left(K^{CB} \right)^{\alpha^{CB}} \left(\Lambda H^{CB} \right)^{1-\alpha^{CB}} \right]^{\xi^{CB}} \left(\frac{E^{CB}}{p} \right)^{1-\xi^{CB}}.$$
(153)

The first order conditions can be compactly rewritten as:

$$i^{D} + i^{W} = \left[\left(1 - \varrho^{CB} \right) \left(i^{W} + i^{D} \right) - i^{D} \right] \left(1 - \xi^{CB} \right) \frac{D^{T}}{E^{CB}},$$
(154)

$$\left(1+\psi_{K}i^{W}\right)r = \left[\left(1-\varrho^{CB}\right)\left(i^{W}+i^{D}\right)-i^{D}\right]\xi^{CB}\alpha^{CB}\frac{D^{T}}{K^{CB}},\tag{155}$$

$$\left(1+\psi_L i^W\right)\left(1+\tilde{\tau}_L\right)w_z^{CB} = \tag{156}$$

$$\left[\left(1-\varrho^{CB}\right)\left(i^{W}+i^{D}\right)-i^{D}\right]\xi^{CB}\left(1-\alpha^{CB}\right)\varkappa_{z}\frac{D^{T}}{H_{z}^{CB}}\left(\frac{a_{z}H_{z}^{CB}}{H^{CB}}\right)^{\kappa},$$
$$\frac{D^{T}}{p}=Az^{CB}\left[\left(K^{CB}\right)^{\alpha^{CB}}\left(\Lambda H^{CB}\right)^{1-\alpha^{CB}}\right]^{\xi^{CB}}\left(\frac{E^{CB}}{p}\right)^{1-\xi^{CB}}.$$
(157)

Note that equation (154) can be rewritten as:

$$i^{W} = \left[\frac{\varrho^{CB} \left(1 - \xi^{CB}\right) \frac{D^{T}}{E^{CB}} + 1}{\left(1 - \varrho^{CB}\right) \left(1 - \xi^{CB}\right) \frac{D^{T}}{E^{CB}} - 1}\right] i^{D}.$$
(158)

Equation (154) has a clear economic interpretation: if the bank increases excess reserves by one (marginal) unit, it decreases the amount of working-capital loans by the same amount, incurring therefore in an opportunity cost equal to $i^D + i^W$. This extra unit of reserves is used in the production of financial services, and increases total demand deposits D^T by its marginal productivity $\left(1 - \xi^{CB}\right) \frac{D^T}{E^{CB}}$. Finally, the increase in total demand deposits will be used to increase working capital loans, and this will lead to an increase in revenues equal to $\left(1 - \varrho^{CB}\right) (i^W + i^D)$, but also implies that these new loans have to be financed via households' demand deposits, that cost i^D . The other conditions have now a straightforward interpretation.

Equation (154) implies that:

$$E^{CB} = \Upsilon^{CB} D^T, \tag{159}$$

where:

$$\Upsilon^{CB} \equiv \frac{\left[\left(1-\varrho^{CB}\right)\left(i^{W}+i^{D}\right)-i^{D}\right]\left(1-\xi^{CB}\right)}{i^{D}+i^{W}}.$$
(160)

Hence, equation (157) can be rewritten as:

$$\frac{D^T}{p} = \Psi^{FB} \left(K^{CB} \right)^{\alpha^{CB}} \left(H^{CB} \right)^{1 - \alpha^{CB}},\tag{161}$$

where:

$$\Psi^{FB} \equiv \left(Az^{CB}\right)^{\frac{1}{\xi^{CB}}} \left(\Upsilon^{CB}\right)^{\frac{1-\xi^{CB}}{\xi^{CB}}}.$$
(162)

Equations (136) and (137), together with the first order conditions (154)-(157) determine in equilibrium the following variables:

$$D^F, D^H, K^{CB}, H_z^{CB}, i^W, i^D.$$
 (163)

Note that, taking (154) into account:

$$\Pi^{CB} = i^{W} D^{F} - i^{D} D^{H} - \left[\left(1 - \varrho^{CB} \right) \left(i^{W} + i^{D} \right) - i^{D} \right] \xi^{CB} D^{T} = 0,$$
(164)

since:

$$(1+\psi_{K}i^{W})rK^{CB} + \sum_{z=1}^{2} (1+\psi_{L}i^{W})(1+\tilde{\tau}_{L})w_{z}^{CB}H_{z}^{CB} = [(1-\varrho^{CB})(i^{W}+i^{D})-i^{D}]\xi^{CB}D^{T}, \quad (165)$$

and, thanks to (154) and (141):

$$\left[\left(1-\varrho^{CB}\right)\left(i^{W}+i^{D}\right)-i^{D}\right]\xi^{CB}D^{T}=i^{W}D^{F}-i^{D}D^{H}.$$
(166)

2.4.2 Bargaining

The general framework for bargaining goes through, so here we repeat those equations only that are specific to this sector. The outside option for the bank if negotiations fail can be written as:

$$\pi^{CB} = -\left(1 + \psi_K i^W\right) r K^{CB},\tag{167}$$

which implies that the value added of successful negotiations is:

$$\tilde{\pi}^{CB} = \left[\left(1 - \varrho^{CB} \right) \left(i^W + i^D \right) - i^D \right] \xi^{CB} D^T - \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \sum_{z=1}^2 w_z^{CB} H_z^{CB}.$$
(168)

The general formulation of the first order condition is the same:

$$\theta_z^{CB} \tilde{\pi}^{CB} \left[H_z^{CB} + \left(w_z^{CB} - \bar{w}_z \right) \frac{\partial H_z^{CB}}{\partial w_z^{CB}} \right] + \left(1 - \theta_z^{CB} \right) \left(w_z^{CB} - \bar{w}_z \right) H_z^{CB} \frac{\partial \tilde{\pi}^{CB}}{\partial w_z^{CB}} = 0.$$
(169)

but the derivative inside is different:

$$\frac{\partial \tilde{\pi}^{CB}}{\partial w_z^{CB}} = \left[\left(1 - \varrho^{CB} \right) \left(i^W + i^D \right) - i^D \right] \xi^{CB} \frac{\partial D^T}{\partial w_z^{CB}} + \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \left(H_z^{CB} + w_z^{CB} \frac{\partial H_z^{CB}}{\partial w_z^{CB}} \right).$$
(170)

where, thanks to (161):

$$\frac{\partial D^{T}}{\partial w_{z}^{CB}} = \frac{\partial D^{T}}{\partial H^{CB}} \frac{\partial H^{CB}}{\partial H_{z}^{CB}} \frac{\partial H_{z}^{CB}}{\partial w_{z}^{CB}}$$

$$= \left(1 - \alpha^{CB}\right) \frac{D^{T}}{H^{CB}} a_{z} \varkappa_{z} \left(\frac{a_{z} H_{z}^{CB}}{H^{CB}}\right)^{\kappa-1} \frac{\partial H_{z}^{CB}}{\partial w_{z}^{CB}}$$

$$= \frac{\left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) H_{z}^{CB}}{\left[\left(1 - \varrho^{CB}\right) \left(i^{W} + i^{D}\right) - i^{D}\right] \xi^{CB}} \epsilon_{z}^{CB}, \qquad (171)$$

where:

$$\epsilon_z^{CB} \equiv \frac{\partial H_z^{CB}}{\partial w_z^{CB}} \frac{w_z^{CB}}{H_z^{CB}}.$$
(172)

Hence:

$$\frac{\partial \tilde{\pi}^{CB}}{\partial w_z^{CB}} = -\left(1 + \psi_L i^W\right) \left(1 + \tilde{\tau}_L\right) H_z^{CB}.$$
(173)

Combining (169) and (173) gets:

$$\theta_z^{CB} \frac{\tilde{\pi}^{CB}}{H_z^{CB}} \left(1 + \frac{w_z^{CB} - \bar{w}_z}{w_z^{CB}} \epsilon_z^{CB} \right) = \left(1 - \theta_z^{CB} \right) \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \left(w_z^{CB} - \bar{w}_z \right). \tag{174}$$

Equation (156) implies that:

$$\frac{\left(1+\psi_{L}i^{W}\right)\left(1+\tilde{\tau}_{L}\right)H^{CB}}{\left[\left(1-\varrho^{CB}\right)\left(i^{W}+i^{D}\right)-i^{D}\right]\xi^{CB}\left(1-\alpha^{CB}\right)} = \left[\frac{\partial D^{T}}{\partial w_{z}^{CB}}-\left(1-\kappa\right)\frac{D^{T}}{H_{z}^{CB}}\frac{\partial H_{z}^{CB}}{\partial w_{z}^{CB}}-\kappa\frac{D^{T}}{H^{CB}}\frac{\partial H^{CB}}{\partial H_{z}^{CB}}\frac{\partial H^{CB}}{\partial w_{z}^{CB}}\right]\frac{\partial H^{CB}}{\partial H_{z}^{CB}}.$$
 (175)

We can now substitute (171), and then (156), finally obtaining:

$$\epsilon_z^{CB} = \left[\left(1 - \alpha^{CB} - \kappa \right) \varkappa_z \left(\frac{a_z H_z^{CB}}{H^{CB}} \right)^{\kappa} + \kappa - 1 \right]^{-1} \tag{176}$$

2.5 International banking sector

There exists a representative, competitive bank that intermediates loans between foreign households and foreign firms, producing financial services using capital, labor and reserves. The bank is owned by the local households. As before, the international bank needs to finance a given fraction of its factor expenditure in advance, before revenues can be cashed in. To finance the foreign demand for loans, our bank issues demand deposit liabilities to foreign households. Hence, total demand deposits are denoted D^{FB} . We assume that the supply of foreign demand deposits (D^*) is exogenous, and D^{FB} depends positively on the interest rate paid by the international bank, denoted i^{FB} :

$$D^{FB} = \left(\frac{i^{FB}}{i}\right)^{\frac{1}{\sigma^{FB}-1}} D^*, \tag{177}$$

where $\sigma^{FB} \in (1, 2)$ so:

$$\frac{dD^{FB}}{di^{FB}} > 0, \quad \frac{d^2 D^{FB}}{\left(di^{FB}\right)^2} < 0.$$
 (178)

Given that foreign firms have access to the international financial market, the foreign recipient of the loans pays an interest rate equal, in equilibrium, to the exogenous international interest rate i. Loans to foreign firms and foreign demand deposits share the same maturity. Loans are extended just prior to production, and then paid off after production. Households deposit funds into the bank just prior to production, and then liquidate the deposit after production.

Foreign demand deposits are associated with financial services. The bank has a technology for converting labor and capital services, and excess reserves, into transaction services:

$$\frac{D^{FB}}{p} = Az^{FB} \left[\left(K^{FB} \right)^{\alpha^{FB}} \left(\Lambda H^{FB} \right)^{1-\alpha^{FB}} \right]^{\xi^{FB}} \left(\frac{E^{FB}}{p} \right)^{1-\xi^{FB}} = Az^{FB} \left(y^{FB} \right)^{\xi^{FB}} \left(\frac{E^{FB}}{p} \right)^{1-\xi^{FB}}, \quad (179)$$

where E^{FB} denote excess reserves:

$$E_t^{FB} = \left(1 - \varrho^{FB}\right) D_t^{FB},\tag{180}$$

and:

$$H^{FB} = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} H_{z}^{FB}\right)^{\kappa}\right]^{\frac{1}{\kappa}}.$$
(181)

Hence, we can solve the production function for D^{FB}/p , getting:

$$\frac{D^{FB}}{p} = \left[\left(1 - \varrho^{FB} \right)^{1 - \xi^{FB}} A z^{FB} \right]^{\frac{1}{\xi^{FB}}} \left(K^{FB} \right)^{\alpha^{FB}} \left(\Lambda H^{FB} \right)^{1 - \alpha^{FB}}, \tag{182}$$

or:

$$\frac{D^{FB}}{p} = \Psi^{FB} \left(K^{FB} \right)^{\alpha^{FB}} \left(\Lambda H^{FB} \right)^{1 - \alpha^{FB}}.$$
(183)

The bank's sources of funds are: interest and principal on loans to foreign firms, plus the resources it received from foreign households at the start of the period. The bank's uses of funds are: principal and interest payments on demand deposits, plus principal on loans, plus gross expenses on labor and capital services. Then, the bank's net source of funds at the end of the period is:

$$\Pi^{FB} = D^{FB} + (1+i) \left(1 - \varrho^{FB}\right) D^{FB} + - \left(1 + i^{FB}\right) D^{FB} - \left(1 - \varrho^{FB}\right) D^{FB} + - \left(1 + \psi_{K} i^{W}\right) r K^{FB} - \left(1 + \psi_{L} i^{W}\right) (1 + \tilde{\tau}_{L}) \sum_{z=1}^{2} w_{z}^{FB} H_{z}^{FB},$$
(184)

that can be compactly rewritten as:

$$\Pi^{FB} = \left[\left(1 - \varrho^{FB} \right) i - i^{FB} \right] D^{FB} + \left(1 + \psi_K i^W \right) r K^{FB} - \left(1 + \psi_L i^W \right) (1 + \tilde{\tau}_L) \sum_{z=1}^2 w_z^{FB} H_z^{FB}.$$
(185)

2.5.1 Profit mazimization

The bank solves the following maximization problem:

$$\max_{\{K^{FB}, H_{z}^{FB}\}} \Pi^{FB} = \left[\left(1 - \varrho^{FB} \right) i - i^{FB} \right] D^{FB} + (186)$$
$$- \left(1 + \psi_{K} i^{W} \right) r K^{FB} - \left(1 + \psi_{L} i^{W} \right) \left(1 + \tilde{\tau}_{L} \right) \sum_{z=1}^{2} w_{z}^{FB} H_{z}^{FB},$$
s.t.
$$\frac{D^{FB}}{p} = \Psi^{FB} \left(K^{FB} \right)^{\alpha^{FB}} \left(\Lambda H^{FB} \right)^{1 - \alpha^{FB}}.$$

The FOCs are (w.r.t. K^{FB}, H_z^{FB}):

$$(1+\psi_K i^W) r = [(1-\varrho^{FB}) i - i^{FB}] \alpha^{FB} \frac{D^{FB}}{K^{FB}}, \qquad (187)$$

$$\left(1+\psi_L i^W\right)\left(1+\tilde{\tau}_L\right)w_z^{FB} = \left[\left(1-\varrho^{FB}\right)i-i^{FB}\right]\left(1-\alpha^{FB}\right)\varkappa_z \frac{D^{FB}}{H_z^{FB}}\left(\frac{a_z H_z^{FB}}{H^{FB}}\right)^\kappa, \quad (188)$$

where:

$$D^{FB} = p \left[\left(1 - \varrho^{FB} \right)^{1-\xi^B} A z^{FB} \right]^{\frac{1}{\xi^B}} \left(K^{FB} \right)^{\alpha^{FB}} \left(\Lambda H^{FB} \right)^{1-\alpha^{FB}}.$$
(189)

Equations (177) and (189), together with the first order conditions (187) and (188), determine in equilibrium the following variables:

$$D^{FB}, K^{FB}, H_z^{FB}, i^{FB}.$$
 (190)

Note that:

$$\Pi^{FB} = 0, \tag{191}$$

since:

$$(1+\psi_{K}i^{W})rK^{FB} + \sum_{z=1}^{2} (1+\psi_{L}i^{W})(1+\tilde{\tau}_{L})w_{z}H_{z}^{FB} = [(1-\varrho^{FB})i-i^{FB}]D^{FB}.$$
 (192)

2.5.2 Bargaining

The general framework for bargaining goes through, so here we repeat those equations only that are specific to this sector. The outside option for the bank if negotiations fail can be written as:

$$\pi^{FB} = -\left(1 + \psi_K i^W\right) r K^{FB},\tag{193}$$

which implies that the value added of successful negotiations for firms is:

$$\tilde{\pi}^{FB} = \left[\left(1 - \varrho^{FB} \right) i - i^{FB} \right] D^{FB} - \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \sum_{j=1}^2 w_j^{FB} H_j^{FB}.$$
(194)

The general formulation of the first order condition is the same:

$$\theta_z^{FB}\tilde{\pi}^{FB}\left[H_z^{FB} + \left(w_z^{FB} - \bar{w}_z\right)\frac{\partial H_z^{FB}}{\partial w_z^{FB}}\right] + \left(1 - \theta_z^{FB}\right)\left(w_z^{FB} - \bar{w}_z\right)H_z^{FB}\frac{\partial\tilde{\pi}^{FB}}{\partial w_z^{FB}} = 0,\tag{195}$$

but the derivative inside is different:

$$\frac{\partial \tilde{\pi}^{FB}}{\partial w_z^{FB}} = \left[\left(1 - \varrho^{FB} \right) i - i^{FB} \right] \frac{\partial D^{FB}}{\partial w_z^{FB}} - \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \left(H_z^{FB} + w_z^{FB} \frac{\partial H_z^{FB}}{\partial w_z^{FB}} \right), \tag{196}$$

where:

$$\frac{\partial D^{FB}}{\partial w_z^{FB}} = \frac{\partial D^{FB}}{\partial H^{FB}} \frac{\partial H^{FB}}{\partial H_z^{FB}} \frac{\partial H_z^{FB}}{\partial w_z^{FB}}
= (1 - \alpha^{FB}) \frac{D^{FB}}{H^{FB}} a_z \varkappa_z \left(\frac{a_z H_z^{FB}}{H^{FB}}\right)^{\kappa-1} \frac{\partial H_z^{FB}}{\partial w_z^{FB}}
= \frac{(1 + \psi_L i^W) (1 + \tilde{\tau}_L) H_z^{FB}}{[(1 - \varrho^{FB}) i - i^{FB}]} \epsilon_z^{FB}.$$
(197)

This implies that:

$$\frac{\partial \tilde{\pi}^{FB}}{\partial w_z^{FB}} = -\left(1 + \psi_L i^W\right) \left(1 + \tilde{\tau}_L\right) H_z^{FB}.$$
(198)

Combining the two yields:

$$\theta_z^{FB} \left(1 + \frac{w_z^{FB} - \bar{w}_z}{w_z^{FB}} \epsilon_z^{FB} \right) \frac{\tilde{\pi}^{FB}}{H_z^{FB}} = \left(1 - \theta_z^{FB} \right) \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i_t^W \right) \left(w_z^{FB} - \bar{w}_z \right). \tag{199}$$

Equation (188) implies that:

$$\frac{\left(1+\psi_{L}i^{W}\right)\left(1+\tilde{\tau}_{L}\right)H^{FB}}{\left[\left(1-\varrho^{FB}\right)i-i^{FB}\right]\left(1-\alpha^{FB}\right)} = \left[\frac{\partial D^{FB}}{\partial w_{z}^{FB}}-\left(1-\kappa\right)\frac{D^{FB}}{H_{z}^{FB}}\frac{\partial H_{z}^{FB}}{\partial w_{z}^{FB}}-\kappa\frac{D^{FB}}{H^{FB}}\frac{\partial H^{FB}}{\partial H_{z}^{FB}}\frac{\partial H^{FB}}{\partial w_{z}^{FB}}\right]\frac{\partial H^{FB}}{\partial H_{z}^{FB}}.$$
 (200)

We can now substitute (197), and then (188) again, finally obtaining:

$$\epsilon_z^{FB} = \left[\left(1 - \alpha^{FB} - \kappa \right) \varkappa_z \left(\frac{a_z H_z^{FB}}{H^{FB}} \right)^{\kappa} + \kappa - 1 \right]^{-1}.$$
 (201)

2.6 Government

The Government budget constraint is:

$$B_t = R_t B_{t-1} + G_t - T_t, (202)$$

where G and T indicate, respectively, total expenses and revenues, while B is government debt.

The Government collects revenues from taxes on the returns on financial assets (A), on profits, and on labour income (H_1 and H_2 are, respectively, resident and non-resident workers, whose wages are w_1 and w_2 , unemployment benefits are \overline{w} ; workers pay taxes at the rate τ_L and firms pay social contributions at the rate $\tilde{\tau}_L$). Furthermore, the government collects taxes on consumption and on imports. Therefore, total revenues in period t amount to:

$$T_{t} = \tau_{K} \left[i_{t} F_{t-1} + (r_{t} - p_{t} \delta_{K}) K_{t-1} + \Pi_{t} + i_{t}^{D} \left(\bar{M}_{t} - \iota_{t} C_{t} \right) \right] + + (\tau_{L} + \tilde{\tau}_{L}) (w_{1,t} H_{1,t} + w_{2,t} H_{2,t}) + + \tau_{L} \bar{w}_{1,t} (1 - H_{1,t}) + \tau_{C} p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_{t}} \right) \right] C_{t} + + t^{M} (1 - \Theta^{*}) N^{*} p_{M}^{*} x^{M}.$$
(203)

where t^M , Θ , N, Θ^* , N^{*}, p_M^* , x^M represent respectively the import tariff (entering into the Government revenues by simplification), the share of domestic varieties that can be traded, the total number of domestic varieties, the share of foreign varieties that can be traded, the total number of foreign varieties, the price of these foreign varieties, the quantity imported.

Government expenditure is composed of unemployment benefits for residents (SUBS), transfers to nonresident workers (TRF), and core expenditure (\bar{G}), where the latter can be further split into other transfers to resident households (TR), public investment in infrastructures (INFR_INV), and general government consumption (GCON). Overall, we have:

$$G_t = SUBS_t + TRF_t + \bar{G}_t, \tag{204}$$

$$SUBS_t = \bar{w}_{1,t} \left(1 - H_{1,t} \right),$$
 (205)

$$TRF_t = TR_t^F \left(\tau_L + \tilde{\tau}_L\right) w_{2,t} H_{2,t}, \qquad (206)$$

$$TR_t = \varrho_1 \bar{G}_t, \tag{207}$$

$$INFR_INV_t = \varrho_2 \bar{G}_t, \tag{208}$$

$$GCON_t = (1 - \varrho_1 - \varrho_2)\bar{G}_t. \tag{209}$$

where $\rho \in (0, 1)$ represents the share of transfers to resident households from core government expenditure. Note that transfers TRF are modelled as a percentage (TR_t^F) of total labour taxes on non-resident workers. The stock of public infrastructures evolves according to the following accumulation equation:

$$INFR_{t} = (1 - \delta_{INFR}) INFR_{t-1} + INFR_{INV_{t}}, \qquad (210)$$

and affects Total Factor Productivity via a purely external effect (see Section 4.1 and Appendix B for further details). Note that δ_{INFR} represents the depreciation rate for public infrastructures.

We further assume that core government expenditure is persistent and depends on the part of the (primary) deficit which excludes core government expenditure, $T_t - (G_t - \bar{G}_t)$:

$$\bar{G}_{t} = \vartheta \bar{G}_{t-1} + (1-\vartheta) d^{LR} \left[T_{t} - \bar{w}_{1,t} \left(1 - H_{1,t} \right) - TR_{t}^{F} \left(\tau_{L} + \tilde{\tau}_{L} \right) w_{2,t} H_{2,t} \right].$$
(211)

This specification of the Government sector implies a zero public debt and deficit in steady state when $d^{LR} = 1$. Otherwise, a value of $d^{LR} > 1$, combined with that of the other variables and parameters in (211), determines the equilibrium level of debt and deficit. Note that the parameter ϑ measures the persistence of core government expenditure.

3 Symmetric equilibrium

In a symmetric equilibrium for all firms in a given sector the prices charged for the differentiated goods and the quantities produced are the same, i.e., $p_j^i = p^i$ and $y_j^i = y^i$, where i = NT, T, M. Furthermore, the equilibrium is characterized by the optimality conditions for households and government.

In Appendix A, first we specialize the analysis of the production sector and labour market to the case of a CES production function, and then we summarize the equilibrium conditions for the various sectors under the case of these CES production functions. The equilibrium conditions are normalized by the exogenous technological progress and by the cohort size, so that we express variables in efficiency terms. For the sake of simplicity, we maintain the previous notation, but now variables are measured in efficiency units.

The set of equilibrium conditions are used in the next section to fully calibrate the model.

4 Calibration and steady state analysis

Due to the complexity of LSM2 and the availability of only 15 years of quarterly observations for Luxembourg, the model cannot be estimated and we have to fully calibrate it. In this section we summarize the calibration procedure for the model parameters, and then discuss the resulting steady state. Appendix B lists all the parameters of LSM, summarizes their meaning, and discusses their calibration in more detail.

4.1 Calibration

We can divide the model parameters into three groups according to how we set their values. The parameters in the first group are set directly to standard values in the DSGE literature. In particular, we fix the subjective discount rate (β) to 0.995, the elasticity of intertemporal substitution to unity (i.e. $\sigma = 1$ which implies that preferences are logarithmic), the weight of capital in the production functions in all sectors (α , α^{CB} and α^{FB}) to 0.36 (the implied capital share in production is 39%), the persistence of core government expenditure (ϑ) to 0.9, the returns to variety to zero (which implies that $\rho = 1$), the elasticity of substitution among intermediate goods to 6 (so that $\mu = 1.2$), the persistence of the stochastic, persistent, but stationary component of productivity to 0.95 and the elasticity of substitution between the two labour types in the CES labour aggregator to 1.5 (so that $\kappa = 1/3$).⁶ We set the relative bargaining power of the unions (θ_z^I , I = T, NT, M, CB, FB) to 0.35 for the tradable and non-tradable intermediate goods producers, to 0.27 for the distributors of imported intermediate goods, and to 0.65 for the domestic and international banks.

We follow Backus, Henriksen, and Storesletten (2008) in setting the depreciation rate on physical capital (δ_K) to 8.5% and on the stock of public infrastructure (δ_{INFR}) to 4.15%. We set the elasticity of the international interest rate with respect to the national debt/GDP ratio (ξ_i) to 0.000742 based on Schmitt-Grohe and Uribe (2004). Following Boldrin, Christiano, and Fisher (2001) we assume that the elasticity of the adjustment cost with respect to the investment-capital ratio is 0.23 (so that $\varsigma = -3.348$).

We set the parameter related to the elasticity of substitution between durables and non-durables in the utility function (v) in order to reproduce an elasticity of substitution equal to 1.5. The percentage of total labour taxes on non-resident workers that is transferred back to non-resident workers (TR_t^F) is chosen to be 0.6. We choose a small value for the fixed cost to enter the market of intermediate good j (ψ_j) and set it equal to 0.00001. The parameter related to the elasticity of TFP with respect to public infrastructure (ϖ) is chosen to be equal to 0.01.

Next, we normalize the foreign aggregate price level (P^*) , the labour-augmenting productivity parameter (Λ) and the parameters augmenting type-1 (a_1) and type-2 (a_2) labour in the labour CES aggregator to unity. We also assume that Luxembourg and the rest of the world are symmetric in terms of the share of non-traded varieties, both Θ and Θ^* are equal to 0.5. We normalize the number of traded varieties to unity, which implies that we set both N and N^{*} equal to 2, again for the sake of symmetry.

We follow Christiano, Motto and Rostagno (2008, 2010) by setting the fraction of the rental and wage bills the firms must finance in advance (ψ_K and ψ_L) to 92% and the required reserves coefficients for both the domestic and international banks (ρ^{CB} and ρ^{FB}) to 2%. We set the parameter related to the elasticity of foreign demand deposits to the interest rate paid by the international bank (σ^{FB}) to 1.5.

For the second parameter group, some values are directly observable or can be estimated. Average life expectancy at birth in Luxembourg was 79.18 years in 2008 (CIA factbook) which implies that the individual survival rate in our model (φ) is 0.987. The average value of net foreign position (\bar{f}) was 85% of GDP at the end of 2007 an 2008 (according to the BcL bulletin). The population growth rate in Luxembourg is 1.2% (data from CIA factbook, year 2008) which implies that η equals to 1.012.

We average depreciation rates for durable goods owned by consumers estimated by the Bureau Economic Analysis over all types of durable goods and set the depreciation rate on durables (δ_D) to 21.7%.

Guarda (1997) estimates the elasticity of substitution between capital and labour in a CES production function to be 1.012 in the tradables sector in Luxembourg (implies that $\lambda = 0.012$). We set the share of type-1 labour in the labour CES aggregator (\varkappa_1) to 0.6 to reflect the fact that approximately 60% of the employed workforce is resident.

We set the tax rates in LSM according to the values reported in *Taxation trends in the EU*, European

⁶Guarda (2000) actually found evidence of complementarity between these labour types in Industry and in Services, but he was using a Translog production functin with gross output (instead of value added) and intermediate consumption. In addition, his sample covered 1984-1996 using unpublished national accounts data prior to the introduction of ESA95. Thus, these past results may not be relevant for the current analysis.

Commission, 2008. In particular, the tax rate on consumption (τ_C) equals to 25.1%. The total average effective tax rate on labour related income is 29.6%, but only 67.9% of this amount is paid by the employee while the remaining part is paid by the employer. Thus, we set the tax rate on labour related income (τ_L) to 20.1% and the social contribution rate on labour related income $(\tilde{\tau}_L)$ to 9.5%. Estimates of the tax rate on capital income (τ_K) are not reported in the mentioned source due to data availability problems, so we take the average effective tax rate on corporate profits as a useful approximation, and set the parameter equal to 29.6%.

The average TFP growth rate (γ) in Luxembourg over 1995-2009, as reported in the Annual Report of the Luxembourg Central Bank (2006, p. 54) was 0.6%.

We use the Overall Trade Restrictiveness Index (Kee, Nicita and Olarreaga, 2009) for the European Union to set the tariffs in the model at 6.6%. The ad-valorem equivalent of all tariff and non-tariff barriers that the European Union imposed against foreign imports was equal to 6.6% in 2006.⁷ However, in Luxembourg 94.5% of all imported goods were originated from countries within the EEA in 2007 and no tariffs were applied on them. Thus, the average effective tariff on imported goods was 0.363%, which is a weighted average of zero and 6.6%, where the weights are the respective import shares. Similarly, 88.2% of all exported goods from Luxembourg in 2007 were sold within the EEA and were exempt from tariffs. The remaining share of exported goods were subject to a tariff rate of 9%, which is the MA-OTRI in 2006 for the European Union. Thus, the effective tariff on exported goods is 1.062%, which is a weighted average of zero and 9%, where the weights are the respective export shares.

In the third group there are fifteen model parameters that we calibrate jointly so that the resulting steady state matches values observed in the data.

The relative weight of durables, non-durables and money holdings kept for transaction services in the utility function (ϕ_C , ϕ_D and ϕ_M) are calibrated in order to reproduce: i) the share of durables consumption in household final consumption expenditure (average annual share between 1995-2008) 0.116, ii) the ratio between nominal consumption of non-durables and the currency stock in the households' hands (the velocity of money) equal to 7 (average between 2006-2009, OECD data), and iii) their sum is normalized to unity. The fixed individual endowment of money (\bar{m}) is calibrated to reproduce the interest rate on demand deposits i^D equal to 1.94% (OECD-STAN data).

The constant and exogenous long-run interest rate equals $\bar{\imath}$ if the country settles down to a net foreign position equal to its steady-state value. We calibrate its value to match the observed net asset foreign position at 85% of GDP in Luxembourg (represented by \bar{f}). The parameter related to the long-run debt/GDP ratio (d^{LR}) is calibrated in order to reproduce the observed debt/GDP ratio of Luxembourg equal to 0.069.

The share of transfers to resident households (ρ_1) and the share of public investment in infrastructures (ρ_2) in core (government) expenditure are calibrated in order to make the model replicate the share of government transfers in total government expenditure (data from OECD annual national accounts, years 2003-2007) and

⁷The 2008 value is slightly lower (5.8%) but the impact is negligible. More importantly, these indices were calculated for goods only. There is ample evidence that ad valorem equivalents of barriers to trade in services are much higher. Accordingly protection faced by Luxembourg exports is underestimated, though most of the exports are to the rest of the EU with no barriers. We decided not to take this into account as no exhaustive data is available for services.
the share of government investment in total government expenditure (data from OECD annual national accounts, years 2003-2007). The replacement ratio of unemployment benefit for domestic workers (REP1) and the replacement ratio of unemployment benefit for foreign workers (REP2), are both expressed as a share of the total gross income of employed domestic workers. These are calibrated in order to replicate a 5% unemployment rate of type-1 workers and a ratio of type-1 to type-2 workers equal to 1.4238. The calibrated parameter values are reported in Appendix B.

The sector specific productivity parameters of the domestic (z^{CB}) and the international (z^{FB}) banks are restricted to be the same across banks and are jointly calibrated to reproduce the observed ratio between interest rates on demand deposits (i^D) and working capital $(i^D + i^W)$ equal to 2 (OECD-STAN data). We also set the two parameters related to the share of demand deposits in the production function of the domestic and international banks (ξ^{CB} and ξ^{CB}) to be the same accross banks and calibrate this common value in order to replicate the observed ratio between demand deposits to households (d^H) and to firms (D^T) equals to 0.07 (OECD-STAN data). We calibrate the value of the foreign demand deposit level (D^*) to reproduce the observed ratio between net export of services (service trade balance) and GDP equal to 46.82% (average of annual values 2005-2008, data from CIA factbook 2008).

Finally, the foreign real output level (Y^*) , the price of imported goods (p_M^*) and the foreign demand deposit level (D^*) are calibrated to match international trade flows. The foreign real output level (Y^*) and the price of imported goods (p_M^*) are calibrated to reproduce the share of goods exports and goods import in GDP equal to 59.5% and 47.8% (average of annual values 2005-2008, data from CIA factbook 2008), respectively. We normalize the weights on imported foreign varieties in the production function in the distribution sector (π_1) to unity. We then calibrate the weights on capital and labor (π_2) to make the model replicate the share of the transport sector value added in GDP. The latter is adjusted to account for the share of international transport services only in GDP, as in Ravn and Mazzenga (2004), giving a value of 2.5% of GDP (average of annual values between 1995-2009, OECD-STAN data).

4.2 Steady state

We now discuss the steady state resulting from the calibration of LSM2. To start with, we can easily recover the national accounting identity:

$$GDP_{t} = p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_{t}} \right) \right] C_{t} + p_{t} I_{t} + GCON_{t} + INFR_INV_{t} + NX_{t},$$

where net trade can be derived from the definition of the balance of payments:

$$NX_{t} = F_{t} - (1 + i_{t}) F_{t-1} + \left[1 - \tau_{L} + TR_{t}^{F} (\tau_{L} + \tilde{\tau}_{L})\right] w_{2,t} H_{2,t}.$$
(212)

The shares of consumption, investment and public expenditure in GDP turn out to be about 38.6%, 28.6% and 15.9%, respectively, while net exports to GDP is about 16.9%. These values are fairly similar to the actual ones for Luxembourg prior to the crisis period. For example, the corresponding shares for the year 2000 derived from national accounts data are, respectively, 40.9%, 23.1%, 15.1% and 20.9%.

GDP can be also decomposed as

$$GDP_{t} = (1 + \tilde{\tau}_{L}) w_{1,t} H_{1,t} + (1 + \tilde{\tau}_{L}) w_{2,t} H_{2,t} + r_{t} K_{t-1} + \Pi_{t} + i_{t}^{D} \left(\bar{M}_{t} - \iota_{t} C_{t} \right) + t^{M} \left(1 - \Theta^{*} \right) N^{*} p_{M}^{*} x_{t}^{M},$$

where $i_t^D (\iota_t C_t - \bar{m})$ represents the opportunity cost of holding money for transaction services. In this case, the respective shares of wages, profits and returns on capital to GDP are about 51.6%, 18.7% and 29.3%. According to national account data for the year 2000, the compensation of employees to GDP ratio was only slightly lower, at 46.3%, while the operating surplus to GDP ratio was about 41.4%.

In terms of production factors, employment of resident workers is about 95% of the labour force, corresponding to an unemployment rate of about 5% to be compared with an actual value of 3.6% in 2000, increasing afterwards, in particular due to the crisis (up to 7.8% in 2010). About 54% of the resident labour force is employed in the banking sectors according to LSM2, which more generally correspond to the services sectors of the economy, rather in line with the value of 48.6% of employment in the services sectors in Luxembourg in 2000. The ratio of resident to non-resident employment is about 1.42, and the wages of the non-resident workers are about 15% lower than those of the resident workers.

Finally, for the public sector, the deficit is very low (due to a comparable level of tax receipts and expenditures) and the public debt is about 7% of GDP, in line with actual values before the crisis.

5 Reforms in the labour and product markets

In this section our aim is to compare the outcome of simulations performed with LSM2 and with LSM. As LSM was mainly focused on the real economy, we replicate exercises targeting reforms in the real sector. We start by assessing the consequences of two measures illustrating well the functioning of the model and its policy relevance: an increase in the replacement rate and a decrease in the mark-up, associated with liberalization in the product market.

For each of the mentioned policy measures, we focus on the effects on a set of aggregate key variables: changes in wages of resident and non-resident workers, in employment of resident and non-resident workers, in unemployment of resident workers, in the total wage bill for resident and non-resident workers, in overall firms' profits, in the private demand components (Consumption, Investment, Net exports), in gross domestic product (GDP), in the government deficit, and in total factor productivity (TFP). We focus on the changes in each variable with respect to its starting value, and use +, ++ and +++ to denote an increase in the range of, respectively, 0-0.5%, 0.5-1% or larger than 1%. The symbols -, - -, and - - - have a similar interpretation for negative changes. We report results on the impact effects of each shock (1y in the tables) and on the subsequent effects due to propagation, up to 20 years after the shock. ⁸

We compare the policy effects with those obtained running similar experiments using LSM, in order to assess whether the inclusion of the banking and re-export sectors changed substantially the model response to real shocks.

⁸More detailed results and findings for other variables are available upon request.

5.1 Higher replacement rate

We consider a permanent increase of 1% in the replacement rate and show how a measure targeted to alleviate the impact of economic recession on workers may have unexpected effects in presence of imperfectly competitive markets.

Looking at effects of our simulations, summarized in Table 1A, it turns out that, in addition to the expected positive income effect for the unemployed, there is also a positive effect on the wage of workers that are still employed. Due to the bargaining structure of the labour market, if the outside option for workers improves, their wage also has to increase. However, higher wages decrease labour demand and lead to a decrease in employment, with the latter effect dominating the former so that the total wage bill actually decreases. In turn, this lowers consumption, which shrinks firm profits and investment, which further reduces demand and gross domestic product (GDP). The only positive effect is on net trade, since lower consumption decreases imports.⁹ Moreover, the higher replacement rate combined with lower employment raises public expenditures for unemployment benefits. Tax receipts decrease due to lower wages, profits and consumption. And the combination of higher expenditures and lower receipts increases the government deficit. It also determines cuts in government investment (infrastructure, but also research and development, education, etc.), which translate into a negative impact on total factor productivity. These responses are very similar to the same shock evaluated with LSM (see Table 1B).¹⁰

5.2 Lower mark-up

We now assess the consequences of a 1% permanent decrease in the mark-up, a policy often considered capable of improving consumer welfare. However, a lower mark-up may lower firm profits, constraining investment and reducing employment. Hence, a-priori it is not clear whether the overall effects of a lower mark-up are beneficial. The results of this simulation conducted with LSM2 are reported in Table 2A.

Lower prices increase real wages and therefore disposable income and consumption. Higher demand stimulates production, but not enough to increase employment in the presence of higher wages. Hence, employment decreases (and unemployment goes up), though the total wage bill increases thanks to the higher real wages. Higher taxes receipts on income and profits improve the public deficit situation, and free resources for public investment, which in turns has a positive effect on productivity. Hence, the overall effects of enhanced competition in the product market are positive. The effects of this policy are again very similar when evaluated with LSM, see Table 2B and Deak et al. (2011) for more details.¹¹

 $^{^{9}}$ Notice that the way net trade is defined is different in LSM2 and LSM. Hence, the result cannot be directly compared for this variable.

¹⁰Responses to the shock (not reported, available upon request) are fairly homogenous across sectors, with increases in the real wages that trigger decreases in employment for both resident and non resident workers, which are in turn generally associated with lower levels of capital stocks.

¹¹The response to the shock is in this case heterogenous across sectors, due to the different market and cost structures. Starting with the tradable and non tradable sectors, reduced mark-up translates into reduced prices, which in turn increases demand, production, labour and capital demand, and real wages. For the domestic Banks the higher production in the other sectors translates into more demand for working capital. Hence, these Banks increase their demand for labour and capital. Finally, for the international Banks there are no changes in demand but their costs of production increase due to higher real wages and

6 Shocks to the banking sectors

In this section we use LSM2 to assess the consequences of a variety of changes in the banking sectors. Our first objective is to illustrate the general equilibrium properties of LSM2. To proceed, we consider a biased growth driven by an increase in the productivity of the banking sector only. Under such circumstances, we expect in general equilibrium that the advantaged sector will attract resources to grow. Hence a combination of macroeconomic growth and reallocation of resources towards the sector fueling this growth. The second objective is to assess the impact of economic policies and exogenous shocks to the banking sector, like decreases in the supply of foreign deposits, increases in the monetary policy rate, increases in the international and domestic banks reserves, and increases in the working capital requirements. The final objective is to assess the robustness of the results to some changes in the structure of the banking sectors. In particular, we assess the consequences of a different bargaining in the labour market between unions and banks, and of a lower share of the banking sector in the economy.

For each shock, we focus first on the effects on a set of key variables at the sectoral level, including wages, employment and total wage bill of resident and non-resident workers, capital, output in the case of the production sectors, and deposits and interest rates for the banking sectors. Then we move at the aggregate level and evaluate the effects on GDP and private demand components (Consumption, Investment, Net exports), government deficit, capital stock and returns on capital, profits, unemployment, wages, employment and total wage bill of resident and non-resident workers, total assets, and TFP.

As in the case of the product and labour market experiments, we focus on the changes in each variable with respect to its starting value, and use +, ++ and +++ to denote an increase in the range of, respectively, 0-0.5%, 0.5-1% or larger than 1%. The symbols -, - -, and - - - have a similar interpretation for negative changes. We report results on the impact effects of each shock (labeled as 1y in the tables) and on the subsequent effects due to propagation, up to 20 years after the shock. ¹²

6.1 General equilibrium properties of LSM2

We start with the productivity of the banking sector, and more generally of the services sector of an economy. This experiment is justified by the uneven distribution of productivity gains across sectors, and fits well the characteristics of an advanced services economy.¹³ As changes in the productivity in one sector will affect the other sectors in general equilibrium, this experiment helps understanding the general equilibrium properties of the model. In the case of Luxembourg, however, there is an additional and more operational justification to such simulation. The country has engaged an ambitious policy in terms of R&D spending in the framework of the 2020 Agenda. Given that the country is already specialized in a cluster of activities related to financial services, most of the results of this effort should fall on the services grouped here in the international banking sector. This is exactly what the first part of this experiment reproduces: a differentiated increase in productivity in favor of the financial services sector.

capital costs. Hence, they reduce their transformation, demand less labour and capital, and need less foreign deposits so that the interest rate of them decreases.

¹²More detailed results and findings for other variables are available upon request.

¹³This contrasts with the traditional hypothesis that productivity gains are larger in industry.

Interestingly, a positive change in one sector may be detrimental to another sector if resources are displaced. These effects are traditional in any general equilibrium framework fitting full employment. Here, the aggregate effects are uncertain, since more productive capital and labour could lower the required amounts of these factors of production, possibly generating unemployment and a drop in investment in the absence of an increase in aggregate demand. We proceed in two steps, starting with technical progress in the international banking only, before comparing with the situation where productivity arises in the domestic banking sector. We indeed expect different general equilibrium effects as one of the inputs of the international banking sectors is foreign deposits.¹⁴

Let us simply consider a permanent increase of 1% in the productivity of the international banks only, whose effects are summarized in Table 3A at the sectoral level, and in Table 3B at the aggregate level.

The increase in productivity would bring to an increase in profits, which is not possible given perfect competition. As it is relatively less costly to produce banking services, resources shift towards this sector. Banks increase their output, therefore hiring more labour and renting more capital, and increasing the demand of international deposits. Hence, there is an increase in the returns on capital, and in the interest rate on international deposits. Concerning wages, the effects of workers' displacement is uncertain. Outside the banking sector, employment decreases, as well as wages. But as a result of bargaining, wages decrease less than expected, and employment decrease more. Thus, there are more workers to be re-employed in the growing sector than under perfect competition on the labour market. Hence, in the banking sectors, the unions are willing to accept an even lower real wage in exchange for a substantial increase in employment.

Therefore, overall in the international banking sector the increase in productivity determines more employment and more total wages, a minor decrease in per capita wages, more capital and a mild increase in the aggregate return on capital, and higher international deposits and interest rate on them.

The increase in the rental and wage bill in the international banking sector requires higher working capital, and hence the demand for credit from domestic banks increases. To match the increased demand, associated with an increase in the related interest rate on domestic deposits, more capital and labour are needed in the domestic banking sector. The former brings the rental cost of capital further up, the latter determines also in this sector a minor decrease in the real wage, accompanied by a more relevant increase in employment and in the total wage bill, for both resident and non resident workers.

In the tradable goods, non tradable goods and distributors sectors, the higher rental rate of capital decreases investment and the capital stock. This in turn lowers the capital per worker and hence labour productivity. There should therefore be a major decrease in the wage, but this effect is attenuated – as already mentioned – by the bargaining between the unions and the firms. Since the decrease in wage is only limited, firms react by shedding employment, with an overall noticeable effect on the total wage bill in all the three sectors. Lower capital and labour lead to a decrease in output, and in turn this brings to lower profits.

Hence, an interesting story emerges. Indeed increases in productivity that are not homogeneous across sectors determine substantial sectoral reallocations of capital and labour, and affect their relative compen-

¹⁴We have also considered the effects of higher productivity in both banking sectors. However, since the size of the international sector is much higher than that of the domestic sector, the effects are very similar to those reported in the first part of this subsection, and therefore we do not discuss them in detail.

sation across sectors. However, changes in factor returns in the disadvantaged sector are contrasted, with labour compensation losing less than expected due to bargaining, but labour demand adjusting more sharply.

The overall effects on the economy of higher productivity in the international banking sector are shown in Table 3B. The increase in capital in the two banking sectors more than compensates the decrease in the production sectors, so that at the aggregate level investment and capital increase, as well as the returns on capital. For employment and the total wage bill a different story holds, as the per capita wage mildly decreases as explained above. The decrease in total profits in the production sectors more than offsets the higher wage bill, so that overall private consumption, and assets, decrease. Net exports instead increase, mostly due to a decrease in the amount of imports. Overall, the effects on GDP are mildly positive.

In terms of fiscal balance, the deficit decreases. A look at the separate behaviour of expenses and receipts reveals that the former decrease, mostly due to lower unemployment, and the latter as well, though to a lesser extent. Lower receipts are due to the major decrease in profits and taxes on them, an effect that dominates the higher payments coming from higher rental and wage bills.

Let us now consider the consequences of higher productivity in the domestic banking sector, starting with the effects within this sector. Results are summarized in Tables 4A and 4B. As in the previous case, with higher productivity profits would go up, which is not possible so that the banks should increase capital and labour.

However, there is now no major increase in working capital needs in the other sectors, and so no request for higher "production" in this domestic banking sector. Due to the higher productivity, the banks can keep production fixed or mildly decrease it even if using smaller amounts of capital and labour.

Lower labour demand enters into the bargaining and lowers the total wage bill, which enter into the loss function of the unions that ask for a small increase in the per capita wage as a compensation, also related to the increased labour productivity. Since capital and labour are complement, capital demand also decreases. Thus, overall the effects on the domestic banking sector of an increase in productivity within this sector are rather negative, with both capital, labour and the total wage bill decreasing, for both resident and non resident workers. The key for this result is the lack of more demand for the output of this sector. This also implies that the interest rate (the cost of working capital) decreases a bit.

The lower cost of working capital is however beneficial for the production sectors. In both the traded, non traded and distributors sectors there is an increase in demand for capital and labour, associated with higher returns on capital and wages, and also higher total wages, profits and output.

The effect on the international banking sector are similarly positive, since this sector also uses the working capital, However, overall the effects are rather small since the size of the domestic banking sector is limited.

At the aggregate level, from Table 4B, the effects are quantitatively limited but qualitatively positive, except for a small decrease in employment in the medium-long run, which is however associated with higher total wages. All the private demand component increase, as well as GDP, and the fiscal deficit is reduced thanks to higher tax receipts on higher profits, and rental and wage bills. Hence, while also in this case there are relevant sectorial differences, higher productivity is overall positive for the economy as a whole.

6.2 Foreign deposits

We now turn to policy simulations and economic shocks affecting the new sectors in LSM. Firstly, a decrease in the supply of foreign deposits can be used to mimic one of the consequences of the recent financial crisis, where many banks and other investors brought large amounts of funds back home.

We consider the consequences of a permanent 1% exogenous decrease in D^* , the foreign deposits used by the international banks. The results are summarized in Tables 5A and 5B.

Starting with the effect in the international banking sector, as a reaction to lower deposits the banks want to decrease also the other inputs, namely, capital and labour. Due to the firms-union bargaining, we end up with a slight increase in real wages accompanied by a substantial decrease in employment. The capital stock also diminishes substantially, and brings down the cost of capital. Overall the activity of this sector shrinks substantially.

The lower cost of capital is however good news for the tradable, non tradable and distributors sectors, who react by increasing the capital demand, and therefore also the labour demand, due to complementarity across inputs. Hence, wages also increase, though mildly since the unions appreciate the increased labour demand, and the overall total wage bill is higher for both resident and non resident workers. The output of three sectors, and the profits, are higher across the entire simulation period. This outcome would be of course different in the presence of other simultaneous shocks affecting directly these sectors, such as an increase of their financing costs.

The wage and rental bills increase substantially in the traded, non-traded and distributors sectors, but not enough to offset the major drop in the international banking sector. This is evident from the aggregate results in Table 5B. Hence, the overall demand for credit, associated with the working capital, shrinks, which means that there is a contraction in the domestic banking sector. Actually, this sector experiences a drop in the demand for both labour and capital, and a small reduction in the activity level, also associated with slightly lower interest rates on deposits.

At the aggregate level, lower interest rates on deposits stimulate consumption even in the presence of a small decrease in the total wage bill (again related to the dominant role of the international banking sector). However, both investment and net exports decrease, as well as Gross Domestic Product. Profits increase, due to the good performance in the sectors that operate under imperfect competition. This increases the tax receipts, but not enough to compensate for the lower receipts associated with lower wage and rental bills. Hence, the deficit deteriorates, which brings in automatic cuts in public expenditures, including productive expenditures that in turn leads to a (limited) reduction in TFP.

6.3 Monetary policy

An increase in the central bank interest rate permits to assess at a disaggregate sectoral level the effects of a change in monetary policy, enhancing the understanding of the aggregate effects. We consider the effects of a tightening of monetary policy, specifically, a permanent 1% increase in the policy rate. The results are summarized in Tables 6A and 6B.

The higher policy rate translates into a higher interest rate paid on the foreign (non euro area) deposits in

the international banking sector. This increases the supply of funds, and therefore also increases the demand for labour and capital. This is associated with an increase in the returns on capital, but with a decrease in the per capita wages since, as explained above, the unions are willing to accept a small increase in the per capita wage in exchange for a major increase in employment, thus maximizing the overall wage bill, for both residents and non residents. Hence, overall the activity of the international banking sector expands substantially.

In principle we should expect that a tightening of monetary policy should translate into lower output and employment, but here there is an additional effect due to the presence of the international banking system. If and only if Luxembourg offers a better remuneration to deposits, there is a marked increase in the activity of the international banking system that more than compensates the contraction of the rest of the economy as a result of the large size of this sector. Hence, the outcome of increases in interest rates in small very open economies specialised in financial transformation can be very specific. However, the positive effects we find can be a bit over-estimated since we do not distinguish between deposits from euro area and the rest of the world.

The three production sectors of the economy, on the other hand, suffer. This is because higher interest rates and higher demand for capital in the investment bank sector make investment more costly, and hence the capital level shrinks. This causes a reduction in labour demand as well, accompanied by lower total wages for both residents and non resident workers. Overall, the output in the all three sectors shrinks substantially. This is a feature to be taken into consideration in the policy debate in highly specialized economies, in the sense that under certain circumstances the development of the advantaged sector can be detrimental to the other sectors.

The domestic banks face a lower demand for working capital by the production sectors, but a much higher one from the international banking sector. At the same time, higher interest rates make it more convenient to postpone consumption and increase savings, so that the supply of deposits increases, so much so that the increased demand can be matched even in the presence of a lower interest rate on deposits paid by the commercial banks. The capital stock and employment also increase in this sector, as well as the rental and wage bills.

At the aggregate level, we get a mixed picture, see Table 6B. Total private consumption, as said, decreases but investment and net exports increase, the latter due mostly to lower imports since production in the intermediate sectors and consumption shrink, and higher export of financial services. Overall the effects on GDP are positive. Both the capital stock and employment increase, as well as the total capital and wage bills. However, profits are reduced, since the sectors in imperfect competition shrink. The effects on the fiscal balance are positive, due to lower expenditures for unemployment subsidies and higher receipts on capital and labour.

The aggregate results we have obtained are fairly different from the standard text-book analysis, which would suggest lower aggregate investment and GDP as a consequence of a tighter monetary policy. The difference is due to the dominant role in the Luxembourg economy of the international banking sector that actually benefits from the higher policy rates. The results for the more traditional production sectors are indeed in line with standard economic theory.

6.4 Banks reserves

We now consider the consequences of an increase in the bank reserves, which can originate either by regulation or by the desire of the banks to be better prepared in case of emergency situations. We start with a 1% point permanent increase in the reserves of the international banks, and then move to the same type of shock for the domestic banks. The results are presented, respectively, in Tables 7A, 7B and 8A, 8B.

Higher reserves for the international banks is equivalent to a drop in available funds to be used as input in the "production" process. Since the production inputs are complementary, the banks also decrease their demand of labour and capital, with subsequent negative effects on the returns on capital and on the total wage bill.

The lower capital cost is good news for the production sectors, and they all react by increasing the demand of capital, and as a consequence of labour. Hence, these sectors expand while the international banking sector shrinks.

The domestic banks are mostly affected through the changes in the working capital, that are positive for the production sectors, negative for the international banks, and overall negative. Lower working capital needs therefore generate a decrease in the interest rate on deposits, but also in the demand for labour and capital, with negative consequences similar to those in the international banking sector, though more limited.

At the aggregate level, see Table 7B, the higher returns on capital have a negative effect on investment, but the lower interest rates on deposits stimulate consumption, notwithstanding a decrease in the total wage bill and partly also thanks to higher profits (that we remind are redistributed to the consumers). The overall effects are however quite negative, with higher unemployment and deficit, and lower GDP.

The effects of higher reserves for the domestic banks are also negative, but the transmission mechanism is rather different, as we will now see. Starting from the domestic bank sector, Table 8A shows, at first sight surprisingly, good results. Higher reserves means lower funds available for lending. However, the demand of working capital by the production sectors and the international banks is basically unchanged. Hence, the interest rate paid for the working capital increases substantially, as well as paid by the banks on deposits, inducing consumers to increase the deposits they offer in order to match the increased needs. The higher level of this "production" input requires to increase also the use of capital and labour, with positive effects on employment and the total wage bill for both residents and non resident workers.

For the other sectors, there is no major credit crunch but a substantial increase in the financing costs, that brings both the international banks and the producers in the traded, non-traded and distribution sectors to shrink both capital, and labour, and production. Hence, the results for these sectors in Table 8A are virtually all minuses.

At the aggregate level, the positive effects on the commercial banks are nearly invisible, due to the overall small size of this sector. The negative effects for the other sectors dominate, with negative consequences on virtually all the aggregate variables reported in Table 8B.

6.5 Working capital requirements

The last experiment we consider is a 1% permanent increase in the working capital, which mimics the general increase in the liquidity needs of the firms experienced during the crisis. The results are reported in Tables 9A and 9B.

Starting this time from the production sectors, we see from Table 9A that the effects are very similar across the traded, non traded and distribution sectors, and all negative. The firms in these sectors reduce the demand for capital and labour, and in turn this has negative effects on the returns on capital and on real wages, and of course on output.

The results for the international bank sector are very similar to those mentioned above, with a generalized deterioration in all the variables.

The effects for the domestic banks are instead positive, since they can charge higher interests on the loans to the firms, and can also pay higher interests on the deposits from consumers in order to match the increased need of working capital. This also leads to higher demand for labour and capital, and to an increase in the total wages for both resident and non resident workers.

At the aggregate level, see Table 9B, the negative consequences in the large international banking and production sectors dominate, determining a generalized and prolonged deterioration in the economic conditions. GDP and all the demand components decrease, as well as employment and the total wage bill, for both the residents and the non resident workers. The fiscal position deteriorates substantially, due to lower tax receipts on both profits and capital and income.

6.6 Robustness analysis

In this section we assess how robust are the results we have obtained to some changes in the structure of the banking sector. First, we consider the monopoly union case, where unions act as a monopolist in the labour market and maximize the total wage bill, taking the labour demand of the firms into account. This corresponds to increasing to unity the bargaining power of the unions in the wage negotiations with the banks. Second, we consider the opposite case, where the union power decreases. Finally, we consider a different calibration of the model, such that the value added of the banking sector in steady state decreases from about 53% to about 39%. We have repeated all the policy experiments discussed in this section for each of the three different scenarios, but here for simplicity we will focus on the effects of an increase in the productivity of the international banking sector, the first experiment we have considered. The results are fairly representative of the consequences that the different scenarios have on the outcome of the other policy changes.

Starting with the case of stronger union power in the banking sector, the results are reported in Tables 10a at the disaggregate sectorial level, and 10b at the aggregate economy level. Comparing Tables 10a and 3a, we see hardly any differences. However, a look at the numbers underlying the plus and minuses in the tables (available upon request), reveals that there is a slightly lower decrease in the real wages in the international banking sector, as a consequence of the stronger union power, and a slightly lower increase in employment, coming from the reaction of the banks to the slightly worse conditions for them in the labour market in

terms of higher wages. At the aggregate level, a comparison of Tables 10b and 3b reveals a similar pattern. The overall effects of the higher productivity in the banking sector remain however positive, with slightly higher GDP (after one year) and investment, notwithstanding the negative displacement effects in the other (tradable and non tradable) sectors of the economy, where employment and wages decrease, as in the baseline case, causing a slight decrease in aggregate consumption.

The effects of lower union power are symmetric, and more evident at the aggregate level. A comparison of Tables 11b and 3b reveals that wages, both per capita and as total wage bill, are now slightly lower, and this determines slight more employment (this effect is only visible from the detailed results, which are available upon request).

Finally, the results for the case of a lower share of the banking sector in the value added are reported in Tables 12a and 12b. A lower share of the banking sector implies that the positive effects in this sector coming from higher productivity are lower, but the negative effects on the other sectors are also lower, so that the aggregate effects are not obvious. A comparison of Tables 3b and 12b (and also 3a and 12a) suggests that the effects are fairly similar in the two cases. An analysis of the actual figures confirms this finding, revealing only a slight increase in aggregate GDP, investment and employment in the presence of a smaller banking sector, but accompanied by slightly lower per capita wages and consumption.

Overall, this robustness analysis suggests that the qualitative results we have obtained as an outcome of the policy experiments are fairly robust to modifications in the size of the banking sector, and to the working of the labour market in this important sector of the Luxembourg economy.

7 Conclusions

In this paper we have developed a structural macroeconometric model for Luxembourg of the NOEM-DSGE type characterized by the presence of the banking and distribution sectors, two key sectors for a small open economy and to understand the consequences of the recent financial crises and some related policy responses.

The model, labeled LSM2 since it is an extension of the LSM model (Luxembourg Structural Model), is characterized by a careful theory based specification of the economy, which is represented by households, government, firms and unions, which interact in the product, labour, financial, banking, and distribution markets.

A properly calibrated version of LSM2 provides useful qualitative insights on the expected consequences of a variety of changes in economic policy, and can also be relevant to assess the effects and propagation of several types of economic shocks. When the shocks or policies take place in the real sectors of the economy, the responses are qualitatively similar to those obtained with LSM.

We have then used LSM2 to assess the consequences of a variety of changes in the banking sectors. We have considered, in turn, increases in the productivity of the banking sector, decreases in the supply of foreign deposits, increases in the monetary policy rate, increases in the international and domestic banks reserves, and increases in the working capital requirements. These experiments shed light on the likely consequences, both at the sectoral and at the aggregate level, of a variety of relevant changes in this dominant sector for the Luxembourg economy.

To conclude, while LSM2 includes a set of specific features of the Luxembourg economy, such as a dual labour market combined with strong union power and the presence of a sizable financial sector, we believe that its general structure can be of general interest for modeling small open economies.

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References

- Adam, F. (2004), "Modelling a Small Open Economy: What is Different? The Case of Luxembourg." STATEC, mimeo.
- [2] Adam, F. (2007), "Cahier de variantes modux", Cahier économique no. xy du STATEC.
- [3] Backus, D., Henriksen, E. and Storesletten, K., 2008, "Taxes and the global allocation of capital," Journal of Monetary Economics, 55(1), 48-61.
- Blanchard Olivier J., 1985. "Debt, Deficits and Finite Horizons". Journal of Political Economy, Vol.93(2), pages 223-247.
- [5] Boldrin, M., Christiano, L.J. and Fisher, J.D.M., 2001, "Habit Persistence, Asset Returns, and the Business Cycle," American Economic Review, 91(1), 149-166.
- [6] Christiano, Lawrence J. and Eichenbaum, Martin, 1995. "Liquidity Effects, Monetary Policy, and the Business Cycle," Journal of Money, Credit and Banking, 27(4), 1113-36.
- [7] Christiano, Lawrence, Roberto Motto and Massimo Rostagno, 2010. "Financial factors in economic fluctuations," Working Paper Series 1192, European Central Bank.
- [8] Christiano, Lawrence, Roberto Motto and Massimo Rostagno, 2008. "Shocks, structures or monetary policies? The Euro Area and US after 2001," Journal of Economic Dynamics and Control, 32(8), 2476-2506.
- [9] Deak, S., Fontagne, L., Maffezzoli, M. and Marcellino, M. (2011) "LSM: A DSGE Model for Luxembourg", Economic Modelling, 28, 2862–2872.
- [10] Guarda, P., 1997. A Production Function for Luxemburg: Estimating a CES Function. Cahiers d'économie du Centre Universitaire de Luxembourg, fascicule XI.
- [11] Guarda, P., 2000. Luxembourg's Cross-Border Workers: Estimating a System of Factor Demands. CREA WP 00-4.
- [12] Guarda, P. (2005), "The Luxembourg block of the multi-country model", in Fagan and Morgan (2005).
- [13] Jermann, U. (1998), "Asset Pricing in Production Economies", Journal of Monetary Economics, 41, 257-275
- [14] Kee, Hiau Looi, Allessandro Nicita and Marcelo Olarreaga, 2009. "Estimating Trade Restrictiveness Indices", Economic Journal, 119(534), 172-99.
- [15] Kim, Jinill, 2004. "What determines aggregate returns to scale?". Journal of Economic Dynamics and Control, vol. 28, pages 1577-94.
- [16] Lockwood, B. (1990), Tax Incidence, Market Power, and Bargaining Structure, Oxford Economic Papers, 42, 187-209.

- [17] Mazzenga Elisabetta and Morten O. Ravn, 1998. "Relative price riddles in international business cycle theory: Are transport costs the explanation?," Economics Working Papers 312, Department of Economics and Business, Universitat Pompeu Fabra.
- [18] Ravn, Morten O. and Mazzenga, Elisabetta, 2004. "International business cycles: the quantitative role of transportation costs," Journal of International Money and Finance, 23(4), 645-671.
- [19] Schmitt-Grohe Stephanie and Martin Uribe, 2004. "Optimal Fiscal and Monetary Policy under Sticky Prices". Vol N.114, pages 198-230, February.
- [20] STATEC, LuxMod, 2006. "A Dynamic Multi-Sector General Equilibrium Model for Luxembourg. Technical Report."

8 Appendix A: Derivation of the symmetric equilibrium of LSM2

In the following subsections first we specialize the analysis of the production sector and labour market to the case of a CES production function, and then we summarize the equilibrium conditions for the various sectors under the case of a CES production function. The equilibrium conditions are normalized by the exogenous technological progress and by the cohort size, so that we express variables in efficiency terms. For the sake of simplicity, we maintain the previous notation, but now variables are measured in efficiency units.

8.1 The nested CES case

For the sake of clarity, we do not distinguish between tradable and non-tradable goods in the intermediate goods sector, but the same production function is assumed in both production processes

$$y = A \left[\alpha k^{\lambda} + (1 - \alpha) \left(\Lambda h \right)^{\lambda} \right]^{\frac{1}{\lambda}}, \qquad (213)$$

$$h = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} h_{z}\right)^{\kappa}\right]^{\overline{\kappa}}.$$
(214)

with $\varkappa_2 = 1 - \varkappa_1$. Note that Λ represents a labour-augmenting productivity parameter. We use this nested CES specification since it clearly distinguishes the elasticity of substitution between aggregate labour and capital, and that between the two types of labours. A few additional comments are in order. First, if $\lambda \to 0$ and $\kappa \to 0$, then both CES aggregators collapse to standard Cobb-Douglas forms:

$$y = Ak^{\alpha} \left(\Lambda h\right)^{1-\alpha}, \tag{215}$$

$$h = (a_1 h_1)^{\varkappa_1} (a_2 h_2)^{\varkappa_2}.$$
(216)

In this case, it is evident that \varkappa_j represents the share of labor income that accrues to type-*j* employment. In general, these parameters remain strictly linked to the distribution of income across different types of workers. Second, in (214) only relative labor productivity matters, i.e. a_1/a_2 . Finally, we allow for a (purely external) effect of the stock of public infrastructure $(INFR_t)$ on the Total Factor Productivity, *A*. In particular, we model *A* as:

$$A = (INFR_t)^{\varpi} \cdot EXOG \cdot PROD, \tag{217}$$

where $0 < \varpi < 1$, EXOG represents exogenous technical progress growing at a constant rate γ , and PROD the stochastic, persistent, but stationary component that drives the real business cycle. We assume that it is

$$\log(PROD_t) = \rho \log(PROD_{t-1}) + \varepsilon_{at}.$$
(218)

where $\rho \in (0, 1)$ measures the persistence of productivity.

Note that:

$$\frac{\partial y}{\partial h} = (\Lambda A)^{\lambda} (1 - \alpha) \left(\frac{h}{y}\right)^{\lambda - 1}, \qquad (219)$$

$$\frac{\partial h}{\partial h_z} = \varkappa_z a_z^{\kappa} \left(\frac{h_z}{h}\right)^{\kappa-1},\tag{220}$$

$$\frac{\partial^2 y}{\partial h^2} = (\lambda - 1) \frac{\partial y}{\partial h} \left[1 - \frac{(1 - \alpha) h^{\lambda}}{\alpha (\Lambda k)^{\lambda} + (1 - \alpha) h^{\lambda}} \right] \frac{1}{h},$$
(221)

$$\frac{\partial^2 h}{\partial h_z^2} = (\kappa - 1) \frac{\partial h}{\partial h_z} \left[1 - \varkappa_z \left(\frac{a_z h_z}{h} \right)^{\kappa} \right] \frac{1}{h_z}.$$
(222)

It follows that the first order conditions of the firm can be written as:

$$\frac{p}{\mu} \left(\Lambda A\right)^{\lambda} \left(1-\alpha\right) \left(\frac{h}{y}\right)^{\lambda-1} \varkappa_{z} a_{z}^{\kappa} \left(\frac{h_{z}}{h}\right)^{\kappa-1} = \left(1+\tilde{\tau}_{L}\right) \left(1+\psi_{L} i_{t}^{W}\right) w_{z}, \tag{223}$$

$$\frac{p}{\mu}A^{\lambda}\alpha\left(\frac{k}{y}\right)^{\lambda-1} = \left(1 + \psi_{K}i_{t}^{W}\right)r.$$
(224)

Recall that the elasticity of labour demand with respect to wages was:

$$\frac{\partial h_z}{\partial w_z} \frac{w_z}{h_z} = \frac{1}{h_z} \frac{p \frac{\partial y}{\partial h_z}}{\frac{1-\mu}{\mu} \frac{p}{y} \left(\frac{\partial y}{\partial h_z}\right)^2 + p \frac{\partial^2 y}{(\partial h_z)^2}}.$$
(225)

We can write:

$$\frac{\partial^2 y}{\left(\partial h_z\right)^2} = \frac{\partial}{\partial h_z} \left(\frac{\partial y}{\partial h}\frac{\partial h}{\partial h_z}\right) = \frac{\partial^2 y}{\partial h^2} \left(\frac{\partial h}{\partial h_z}\right)^2 + \frac{\partial y}{\partial h}\frac{\partial^2 h}{\partial h_z^2}$$
(226)

$$= \left[\frac{\lambda-1}{h} - \frac{\lambda-1}{y}\frac{\partial y}{\partial h}\right]\frac{\partial y}{\partial h_z}\frac{\partial h}{\partial h_z} + \left[\frac{\kappa-1}{h_z} - \frac{\kappa-1}{h}\frac{\partial h}{\partial h_z}\right]\frac{\partial y}{\partial h_z},$$
(227)

which implies:

$$\epsilon_z \equiv \frac{\partial h_z}{\partial w_z} \frac{w_z}{h_z} = \frac{1}{h_z} \frac{\frac{\partial y}{\partial h_z}}{\frac{1-\mu}{\mu} \frac{1}{y} \left(\frac{\partial y}{\partial h_z}\right)^2 + \frac{\partial^2 y}{(\partial h_z)^2}}$$
(228)

$$= \left[\left(\frac{1 - \mu \lambda}{\mu} \frac{\partial y}{\partial h} \frac{h}{y} + \lambda - \kappa \right) \frac{\partial h}{\partial h_z} \frac{h_z}{h} + \kappa - 1 \right]^{-1}.$$
 (229)

The production function for the transport services used in the importing sector is similar, but not identical:

$$y^{M} = \pi_{2} \left[\alpha \left(k^{M} \right)^{\lambda} + (1 - \alpha) \left(\Lambda h^{M} \right)^{\lambda} \right]^{\frac{1}{\lambda}}, \qquad (230)$$

where π_2 is a constant productivity parameter. Also the detivation of the elasticity of labor demand is sligtly different:

$$\epsilon_{z}^{M} = \frac{1}{h_{z}^{M}} \frac{\left[p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}\right]\frac{\partial y^{M}}{\partial h_{z}^{M}}}{\frac{1 - \mu}{\mu}\frac{p^{M}}{y^{M}}\left(\frac{\partial y^{M}}{\partial h_{z}^{M}}\right)^{2} + \left[p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}\right]\frac{\partial^{2}y^{M}}{(\partial h_{z}^{M})^{2}}}{\left(\left\{\left[\frac{1 - \mu}{\mu}\frac{p^{M}}{p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}} - \lambda + 1\right]\frac{h^{M}}{y^{M}}\frac{\partial y^{M}}{\partial h^{M}} + \lambda - \kappa\right\}\times\right)^{-1}\right)^{-1} \left(\left(\left\{\left[\frac{1 - \mu}{\mu}\frac{p^{M}}{p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}} - \lambda + 1\right]\frac{h^{M}}{y^{M}}\frac{\partial y^{M}}{\partial h^{M}} + \lambda - \kappa\right\}\times\right)^{-1}\right)^{-1}\right)$$

$$\left(\left(\left(\frac{1 - \mu}{\mu}\frac{p^{M}}{p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}} - \lambda + 1\right)\frac{h^{M}}{y^{M}}\frac{\partial y^{M}}{\partial h^{M}} + \lambda - \kappa\right)\right)^{-1}\right)^{-1}\right)^{-1}$$

$$\left(\left(\frac{1 - \mu}{\mu}\frac{p^{M}}{p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}} - \lambda + 1\right)\frac{h^{M}}{y^{M}}\frac{\partial y^{M}}{\partial h^{M}} + \lambda - \kappa\right)\right)^{-1}\right)^{-1}$$

$$\left(\frac{1 - \mu}{\mu}\frac{p^{M}}{p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}} - \lambda + 1\right)\frac{h^{M}}{y^{M}}\frac{\partial y^{M}}{\partial h^{M}} + \lambda - \kappa\right)^{-1}$$

8.2 Households

The key equations for the Household sector of LSM2 are:

$$C_{t+1} = \eta \mathcal{E}_{t+1} \left(C_t - \frac{\eta - \varphi}{\eta} \frac{A_t}{\zeta_t - \mathcal{Z}_t} \right), \tag{232}$$

$$D_t = \xi_t C_t, \tag{233}$$

$$M_t = \iota_t C_t, \tag{234}$$

$$A_t = R_t A_{t-1} + \mathcal{H}_t - \mathcal{Z}_t C_t, \tag{235}$$

$$\mathcal{H}_t = (1 - \tau_K) \left(i_t^D \bar{M}_t + \Pi_t \right) +$$

$$(1 - \tau_L) \left[w_{1,t} H_{1,t} + \bar{w}_{1,t} \left(1 - H_{1,t} \right) \right] + \varrho_1 \bar{G}_t, \tag{236}$$

were:

$$\zeta_t = \mathcal{Z}_t + \mathcal{E}_{t+1} \frac{\varphi}{R_{t+1}} \zeta_{t+1}, \tag{237}$$

$$\xi_{t} = \left\{ \frac{\phi_{C}}{\phi_{D}} \left[\varkappa_{t}^{d} - \varkappa_{t+1}^{d} \frac{1 - \delta_{D}}{R_{t+1} \frac{p_{t}}{p_{t+1}}} \right] \right\}^{\frac{1}{\nu - 1}},$$
(238)

$$\iota_t = \left(\frac{\phi_C}{\phi_M} \frac{1 - \tau_K}{1 + \tau_C} i_t^D\right)^{\frac{1}{\upsilon - 1}} p_t,\tag{239}$$

$$\mathcal{E}_{t+1} = \left\{ \left[\frac{\phi_C + \phi_D \xi_{t+1}^{\upsilon} + \phi_M \iota_{t+1}^{\upsilon}}{\phi_C + \phi_D \xi_t^{\upsilon} + \phi_M \iota_t^{\upsilon}} \right]^{\frac{1-\upsilon-\sigma}{\upsilon}} \beta R_{t+1} \frac{p_t}{p_{t+1}} \right\}^{\frac{1}{\sigma}},$$
(240)

$$\mathcal{Z}_t = (1 + \tau_C) p_t \left[1 + \varkappa_t^d \left(\xi_t - \frac{1 - \delta_D}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_t} \right) \right] + (1 - \tau_K) i_t^D \iota_t.$$
(241)

8.3 Asset Stock

The key equations for the Asset Stock sector of LSM2 are:

$$F_t = A_t - B_t - p_t \nu_t K_t, \tag{242}$$

$$K_t = \left[1 - \delta_K + \frac{\Xi_1}{\varsigma} \left(\frac{I_t}{K_{t-1}}\right)^{\varsigma} + \Xi_2\right] K_{t-1}, \qquad (243)$$

$$\nu_t = \frac{1}{\Xi_1} \left(\frac{I_t}{K_{t-1}} \right)^{1-\zeta}, \tag{244}$$

$$p_t \nu_t = \frac{(1 - \tau_K) r_{t+1} + p_{t+1} \left(\tau_K \delta_K - \frac{I_{t+1}}{K_t} \right)}{R_{t+1}}$$
(245)

$$+\frac{p_{t+1}\nu_{t+1}\left[1-\delta_K+\frac{\Xi_1}{\varsigma}\left(\frac{I_{t+1}}{k_t}\right)^{\varsigma}+\Xi_2\right]}{R_{t+1}}.$$
(246)

8.4 Intermediate goods sector

8.4.1 Non-tradable goods

The key equations for the non-tradable goods sector and associated labour market are (we drop the time index for the sake of simplicity):

$$y^{NT} = A \left[\alpha (k^{NT})^{\lambda} + (1 - \alpha) \left(\Lambda h^{NT} \right)^{\lambda} \right]^{\frac{1}{\lambda}}, \qquad (247)$$

$$\mu \left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right) w_z^{NT} = p^{NT} (1 - \alpha) (A\Lambda)^\lambda \varkappa_z a_z^\kappa \left(\frac{y^{NT}}{h^{NT}}\right)^{1 - \lambda} \left(\frac{h^{NT}}{h_z^{NT}}\right)^{1 - \kappa}, \tag{248}$$

$$\mu \left(1 + \psi_L i^W\right) r = p^{NT} \alpha A^\lambda \left(\frac{y^{NT}}{k^{NT}}\right)^{1-\lambda}, \tag{249}$$

$$p^{NT} = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{y^{NT}}{Y}\right)^{\frac{1-\mu}{\mu}} p, \qquad (250)$$

$$\theta_z^{NT} \left(1 + \frac{w_z^{NT} - \bar{w}_z}{w_z^{NT}} \epsilon_z^{NT} \right) \frac{\tilde{\pi}^{NT}}{h_z^{NT}} = \left(1 - \theta_z^{NT} \right) \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right) \left(w_z^{NT} - \bar{w}_z \right), \tag{251}$$

$$\pi^{NT} \equiv \left(1 - \frac{1}{\mu}\right) p^{NT} y^{NT} - \psi.$$
(252)

$$h^{NT} = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} h_{z}^{NT}\right)^{\kappa}\right]^{\frac{1}{\kappa}},$$
(253)

$$\epsilon_{z} = \left\{ \begin{bmatrix} \frac{1-\lambda\mu}{\mu} \left(1-\alpha\right) \left(A\Lambda \frac{h^{NT}}{y^{NT}}\right)^{\lambda} + \lambda - \kappa \end{bmatrix} \varkappa_{z} \left(\frac{a_{z}h_{z}^{NT}}{h^{NT}}\right)^{\kappa} + \right\}^{-1}, \qquad (254)$$

$$\tilde{\pi}^{NT} = p^{NT} y^{NT} - (1 + \tilde{\tau}_L) \left(1 + \psi_L i^W \right) \sum_{z=1}^2 w_z^{NT} h_z^{NT}.$$
(255)

8.4.2 Tradable goods

The key equations for the tradable goods sector and associated labour market are (we drop the time index for the sake of simplicity):

$$y^{T} = A \left[\alpha(k^{T})^{\lambda} + (1 - \alpha) \left(\Lambda h^{T} \right)^{\lambda} \right]^{\frac{1}{\lambda}},$$
(256)

$$\mu \left(1 + \tilde{\tau}_L\right) \left(1 + \psi_L i^W\right) w_z^T = p^T (1 - \alpha) (A\Lambda)^{\lambda} \varkappa_z a_z^{\kappa} \left(\frac{y^T}{h^T}\right)^{1 - \lambda} \left(\frac{h^T}{h_z^T}\right)^{1 - \kappa}, \qquad (257)$$

$$\mu \left(1 + \psi_L i^W \right) r = p^T \alpha A^\lambda \left(\frac{y^T}{k^T} \right)^{1-\lambda}, \qquad (258)$$

$$p^T = p^H = p^F, (259)$$

$$p^{H} = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s^{H}y^{T}}{Y}\right)^{\frac{1-\mu}{\mu}} p, \qquad (260)$$

$$p^{F} = \left(1 - t^{F}\right) \left(\mathcal{N}^{*}\right)^{\frac{\rho - \mu}{\mu}} \left(\frac{s^{F} y^{T}}{Y^{*}}\right)^{\frac{1 - \mu}{\mu}} p^{*},$$
(261)

$$\theta_z^T \left(1 + \frac{w_z^T - \bar{w}_z}{w_z^T} \epsilon_z^T \right) \frac{\tilde{\pi}^T}{h_z^T} = \left(1 - \theta_z^T \right) \left(1 + \tilde{\tau}_L \right) \left(1 + \psi_L i^W \right) \left(w_z^T - \bar{w}_z \right), \tag{262}$$

$$\pi^T \equiv \left(1 - \frac{1}{\mu}\right) p^T y^T - \psi.$$
(263)

$$s^F = 1 - s^H, (264)$$

$$h^{T} = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} h_{z}^{T}\right)^{\kappa}\right]^{\frac{1}{\kappa}},$$
(265)

$$\epsilon_{z} = \left\{ \begin{bmatrix} \frac{1-\lambda\mu}{\mu} \left(1-\alpha\right) \left(A\Lambda \frac{h^{T}}{y^{T}}\right)^{\lambda} + \lambda - \kappa \end{bmatrix} \varkappa_{z} \left(\frac{a_{z}h_{z}^{T}}{h^{T}}\right)^{\kappa} \right\}^{-1}, \qquad (266)$$

$$\tilde{\pi}^{T} = p^{T} y^{T} - (1 + \tilde{\tau}_{L}) \left(1 + \psi_{L} i^{W} \right) \sum_{s=1}^{2} w_{s}^{T} h_{s}^{T}.$$
(267)

8.4.3 Distributors of imported intermediate goods

For the distributors we have (we drop the time index for the sake of simplicity):

$$y^{M} = \pi_{2} \left[\alpha \left(k^{M} \right)^{\lambda} + (1 - \alpha) \left(\Lambda h^{M} \right)^{\lambda} \right]^{\frac{1}{\lambda}},$$
(268)

$$\left[p^{M} - \mu \left(1 + t^{M}\right) \frac{p_{M}^{*}}{\pi_{1}}\right] (1 - \alpha) (\pi_{2}\Lambda)^{\lambda} \varkappa_{z} a_{z}^{\kappa} \left(\frac{y^{M}}{h^{M}}\right)^{1 - \lambda} \left(\frac{h^{M}}{h_{z}^{M}}\right)^{1 - \kappa} = \mu \left(1 + \tilde{\tau}_{L}\right) \left(1 + \psi_{L} i^{W}\right) w_{z}^{M},$$
(269)

$$\left[p^{M} - \mu\left(1 + t^{M}\right)\frac{p_{M}^{*}}{\pi_{1}}\right]\alpha\pi_{2}^{\lambda}\left(\frac{y^{M}}{k^{M}}\right)^{1-\lambda} = \mu\left(1 + \psi_{K}i^{W}\right)r,$$
(270)

$$p^{M} = p^{M,H} = p^{M,F}, (271)$$

$$p^{M,H} = \mathcal{N}^{\frac{\rho-\mu}{\mu}} \left(\frac{s^{M,H}y^M}{Y}\right)^{\frac{1-\mu}{\mu}} p, \qquad (272)$$

$$p^{M,F} = (1 - t^F) \left(\mathcal{N}^*\right)^{\frac{\rho - \mu}{\mu}} \left(\frac{s^{M,F}y^M}{Y^*}\right)^{\frac{1 - \mu}{\mu}} p^*,$$
(273)

$$\theta_z^M \left(1 + \frac{w_z^M - \bar{w}_z}{w_z^M} \epsilon_z^M \right) \frac{\tilde{\pi}^M}{h_z^M} = (1 + \tilde{\tau}_L) \left(1 + \psi_L i^W \right) \left(1 - \theta_z^M \right) \left(w_z^M - \bar{w}_z \right), \tag{274}$$

$$\pi^M = \left(1 - \frac{1}{\mu}\right) p^M y^M - \psi. \tag{275}$$

$$s^{M,F} = 1 - s^{M,H}, (276)$$

$$h^{M} = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} h_{z}^{M}\right)^{\kappa}\right]^{\frac{1}{\kappa}}, \qquad (277)$$

$$\epsilon_{z} = \left(\begin{array}{c} \left\{ \left[\frac{1-\mu}{\mu} \frac{p^{M}}{p^{M}-\mu(1+t^{M})\frac{p_{M}^{*}}{\pi_{1}}} - \lambda + 1 \right] (1-\alpha) \left(\pi_{2}\Lambda \frac{h^{M}}{y^{M}}\right)^{\lambda} + \lambda - \kappa \right\} \times \\ \varkappa_{z} \left(\frac{a_{z}h_{z}^{M}}{h^{M}} \right)^{\kappa} + \kappa - 1 \end{array} \right)^{-1}, \quad (278)$$

$$\tilde{\pi}^{M} = \left[p^{M} - \mu \left(1 + t^{M} \right) \frac{p_{M}^{*}}{\pi_{1}} \right] y^{M} - \left(1 + \tilde{\tau}_{L} \right) \left(1 + \psi_{L} i^{W} \right) \sum_{z=1}^{2} w_{z}^{M} h_{z}^{M}.$$
(279)

8.5 Banking

8.5.1 Domestic banks

For the domestic banking sector we have (we drop the time index for the sake of simplicity):

$$D^H = \bar{M} - M = \bar{M} - \iota C, \qquad (280)$$

$$D^{F} = \psi_{K} r K + \psi_{L} \sum_{z=1}^{2} w_{z} H_{z}, \qquad (281)$$

$$\frac{D^T}{p} = \left(Az^{CB}\right)^{\frac{1}{\xi^{CB}}} \left(\Upsilon^{CB}\right)^{\frac{1-\xi^{CB}}{\xi^{CB}}} \left(K^{CB}\right)^{\alpha^{CB}} \left(\Lambda H^{CB}\right)^{1-\alpha^{CB}},\tag{282}$$

$$i^{W} = \left[\frac{\varrho^{CB} \left(1 - \xi^{CB}\right) \frac{D^{T}}{E^{r}} + 1}{\left(1 - \varrho^{CB}\right) \left(1 - \xi^{CB}\right) \frac{D^{T}}{E^{r}} - 1}\right] i^{D},$$
(283)

$$(1 + \psi_L i^W) (1 + \tilde{\tau}_L) w_z^{CB} = \left[(1 - \varrho^{CB}) (i^W + i^D) - i^D \right] \xi^{CB} (1 - \alpha^{CB}) \varkappa_z \frac{D^T}{H_z^{CB}} \left(\frac{a_z H_z^{CB}}{H^{CB}} \right)^{\kappa},$$
(284)

$$\left(1+\psi_{K}i^{W}\right)r = \left[\left(1-\varrho^{CB}\right)\left(i^{W}+i^{D}\right)-i^{D}\right]\xi^{CB}\alpha^{CB}\frac{D^{T}}{K^{CB}},\tag{285}$$

$$\theta_z^{CB} \frac{\tilde{\pi}^{CB}}{H_z^{CB}} \left(1 + \frac{w_z^{CB} - \bar{w}_z}{w_z^{CB}} \epsilon_z^{CB} \right) = \left(1 - \theta_z^{CB} \right) \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \left(w_z^{CB} - \bar{w}_z \right), \tag{286}$$

$$\pi^{FB} = 0. \tag{287}$$

$$D^T = D^H + D^F, (288)$$

$$H^{CB} = \left[\sum_{z=1}^{2} \varkappa_{z} \left(a_{z} H^{CB}_{z,t}\right)^{\kappa}\right]^{\frac{1}{\kappa}}, \qquad (289)$$

$$\Upsilon^{CB} = \frac{\left[\left(1 - \varrho^{CB}\right)\left(i^{W} + i^{D}\right) - i^{D}\right]\left(1 - \xi^{CB}\right)}{i^{D} + i^{W}},\tag{290}$$

$$E^{CB} = D^H - \varrho^{CB} D^T, (291)$$

$$\epsilon_z^{CB} = \left[\left(1 - \alpha^{CB} - \kappa \right) a_z \varkappa_z \left(\frac{a_z H_z^{CB}}{H^{CB}} \right)^{\kappa - 1} + \kappa - 1 \right]^{-1}, \tag{292}$$

$$\tilde{\pi}^{CB} = i^{W} D^{F} - i^{D} D^{H} - \left(1 + \psi_{L} i^{W}\right) \left(1 + \tilde{\tau}_{L}\right) \sum_{z=1}^{2} w_{z}^{CB} H_{z}^{CB}.$$
(293)

8.5.2 International banks

For the international banking sector we have:

$$D^{FB} = \left(\frac{i^{FB}}{i}\right)^{\frac{1}{\sigma^{FB}-1}} D^*,$$
(294)

$$\frac{D^{FB}}{p} = \left[\left(1 - \varrho^{FB}\right)^{1 - \xi^{FB}} A z^{FB} \right]^{\frac{1}{\xi^{FB}}} \left(K^{FB}\right)^{\alpha^{FB}} \left(\Lambda H^{FB}\right)^{1 - \alpha^{FB}},\tag{295}$$

$$\left(1+\psi_L i^W\right)\left(1+\tilde{\tau}_L\right)w_z^{FB} = \left[\left(1-\varrho^{FB}\right)i-i^{FB}\right]\left(1-\alpha^{FB}\right)\varkappa_z \frac{D^{FB}}{H_z^{FB}}\left(\frac{a_z H_z^{FB}}{H^{FB}}\right)^{\kappa},\tag{296}$$

$$(1+\psi_K i^W) r = \left[\left(1-\varrho^{FB}\right) i - i^{FD} \right] \alpha^{FB} \frac{D^{FB}}{K^{FB}},$$
(297)

$$\theta_z^{FB} \left(1 + \frac{w_z^{FB} - \bar{w}_z}{w_z^{FB}} \epsilon_z^{FB} \right) \frac{\tilde{\pi}^{FB}}{H_z^{FB}} = (1 + \tilde{\tau}_L) \left(1 + \psi_L i^W \right) \left(1 - \theta_z^{FB} \right) \left(w_z^{FB} - \bar{w}_z \right), \tag{298}$$

$$\Pi_t^{FB} = 0 \tag{299}$$

$$H_t^{FB} = \left[\sum_{z=1}^2 \varkappa_z \left(a_z H_{z,t}^{FB}\right)^\kappa\right]^{\frac{1}{\kappa}},\tag{300}$$

$$\epsilon_z^{FB} = \left[\left(1 - \alpha^{FB} - \kappa \right) a_z \varkappa_z \left(\frac{a_z H_z^{FB}}{H^{FB}} \right)^{\kappa - 1} + \kappa - 1 \right]^{-1}, \tag{301}$$

$$\tilde{\pi}^{FB} = \left[\left(1 - \varrho^{FB} \right) i - i^{FB} \right] D^{FB} - \left(1 + \psi_L i^W \right) \left(1 + \tilde{\tau}_L \right) \sum_{j=1}^2 w_j^{FB} H_j^{FB}.$$
(302)

8.6 Government

The key equations for the Government sector are:

$$B_t = R_t B_{t-1} + G_t - T_t, (303)$$

$$T_{t} = \tau_{K} \left[i_{t} F_{t-1} + (r_{t} - p_{t} \delta_{K}) K_{t-1} + \Pi_{t} + i_{t}^{D} \left(\bar{M} - \iota_{t} C_{t} \right) \right] + + (\tau_{L} + \tilde{\tau}_{L}) \left(w_{1,t} H_{1,t} + w_{2,t} H_{2,t} \right) + + \tau_{L} \bar{w}_{1,t} \left(1 - H_{1,t} \right) + \tau_{C} p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_{t}} \right) \right] C_{t} + + t^{M} \left(1 - \Theta^{*} \right) N^{*} p_{M}^{*} x_{t}^{M},$$
(304)

$$+t^{TT}(1-\Theta^{\dagger})N^{T}p_{M}^{T}x_{t}^{TT},$$

$$G_{t} = SUBS_{t} + TRF_{t} + \bar{G}_{t},$$

$$(304)$$

$$(305)$$

$$SUBS_t = \bar{w}_{1,t} \left(1 - H_{1,t} \right),$$
(306)

$$TRF_t = TR_t^F \left(\tau_L + \tilde{\tau}_L\right) w_{2,t} H_{2,t},\tag{307}$$

$$TR_t = \varrho_1 \bar{G}_t, \tag{308}$$

$$INFR_INV_t = \varrho_2 \bar{G}_t, \tag{309}$$

$$GCON_t = (1 - \varrho_1 - \varrho_2)\,\bar{G}_t,\tag{310}$$

$$INFR_{t} = (1 - \delta_{INFR}) INFR_{t-1} + INFR_{INV_{t}}, \qquad (311)$$

$$\bar{G}_{t} = \vartheta \bar{G}_{t-1} + (1-\vartheta) d^{LR} \begin{bmatrix} T_{t} - \bar{w}_{1,t} (1-H_{1,t}) \\ -TR_{t}^{F} (\tau_{L} + \tilde{\tau}_{L}) w_{2,t} H_{2,t} \end{bmatrix},$$
(312)

$$\bar{w}_{1,t}^j = rep_1^j GROSSINC_t, \tag{313}$$

$$\bar{w}_{2,t}^j = rep_2^j GROSSINC_t, \tag{314}$$

where:

$$GROSSINC_t = w_{1,t}H_{1,t} + \bar{w}_{1,t}(1 - H_{1,t}) + r_t K_{t-1} + \Pi_t.$$
(315)

The last two equations determine unemployment benefits as a function of net income.

8.7 Aggregation

The aggregate variables are given by:

$$Y = \mathcal{N}^{\rho-\mu} \left[\Theta N \left(y^{NT} \right)^{\frac{1}{\mu}} + (1 - \Theta) N \left(y^{H} \right)^{\frac{1}{\mu}} + (1 - \Theta^{*}) N^{*} \left(y^{M,H} \right)^{\frac{1}{\mu}} \right]^{\mu},$$
(316)

$$p = \mathcal{N}^{\mu-\rho} \begin{bmatrix} \Theta N \left(p^{NT} \right)^{\frac{1}{1-\mu}} + (1-\Theta) N \left(p^{H} \right)^{\frac{1}{1-\mu}} \\ + (1-\Theta^{*}) N^{*} \left(p^{M,H} \right)^{\frac{1}{1-\mu}} \end{bmatrix}^{-1}, \qquad (317)$$

$$H_{z} = \left[\Theta h_{z}^{NT} + (1 - \Theta) h_{z}^{T}\right] N + (1 - \Theta^{*}) N^{*} h_{z}^{M} + H_{z}^{CB} + H_{z}^{FB},$$
(318)

$$w_{z} = \frac{\left[\Theta h_{z}^{NT} w_{z}^{NT} + (1 - \Theta) h_{z}^{T} w_{z}^{T}\right] N + (1 - \Theta^{*}) N^{*} h_{z}^{M} w_{z}^{M} + H_{z}^{CB} w_{z}^{CB} + H_{z}^{FB} w_{z}^{FB}}{H_{z}}, \qquad (319)$$

$$K = \left[\Theta k^{NT} + (1 - \Theta) k^{T}\right] N + (1 - \Theta^{*}) N^{*} k^{M} + K^{CB} + K^{FB}, \qquad (320)$$

 $\Pi = \left[\Theta \pi^{NT} + (1 - \Theta) \pi^T\right] \mathbf{N} + (1 - \Theta^*) \mathbf{N}^* \pi^M.$ (321)

8.8 Numeraire

We set the numeraire as the price of the non-traded goods:

$$p^{NT} = 1.$$
 (322)

8.9 Exogenous variables

The following variables are treated as exogenous in LSM:

$$R_t \equiv 1 + (1 - \tau_K) i_t, \tag{323}$$

$$i_t = \bar{\imath} + \xi_i \left[\exp\left(\bar{f} - \frac{F_t}{GDP_t}\right) - 1 \right] + \varepsilon_{it}, \qquad (324)$$

$$A_t = A_0 \left(INFR_t \right)^{\varpi} \cdot PROD_t, \tag{325}$$

$$\log(PROD_t) = \rho \log(PROD_{t-1}) + \varepsilon_{at}.$$
(326)

8.10 Variables of particular interest

Finally, we develop the equations for some variables of interest. Let's start with the accumulation equation for wealth:

$$A_t = R_t A_{t-1} + \mathcal{H}_t - \mathcal{Z}_t C_t. \tag{327}$$

Substitute the definitions of A_t , \mathcal{H}_t , and \mathcal{Z}_t :

$$F_{t} + B_{t} + p_{t}\nu_{t}K_{t} = R_{t}\left(F_{t-1} + B_{t-1} + p_{t-1}\nu_{t-1}K_{t-1}\right) + (1 - \tau_{K})\left(i_{t}^{D}\bar{M}_{t} + \Pi_{t}\right) + (1 - \tau_{L})\left[w_{1,t}H_{1,t} + \bar{w}_{1,t}\left(1 - H_{1,t}\right)\right] + \varrho_{1}\bar{G}_{t} + \left[\left(1 + \tau_{C}\right)p_{t}\left[1 + \varkappa_{t}^{d}\left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi}\frac{\xi_{t-1}}{\mathcal{E}_{t}}\right)\right] + (1 - \tau_{K})i_{s}^{D}\iota_{s}\right]C_{t}.$$
 (328)

Taking the following no-arbitrage conditon into account:

$$p_{t-1}\nu_{t-1}K_{t-1} = p_t \frac{\left[(1 - \tau_K) \frac{r_t}{p_t} + \tau_K \delta_K \right] K_{t-1} - I_t + \nu_t K_t}{R_t},$$
(329)

leads us to:

$$F_{t} + B_{t} = R_{t} \left(F_{t-1} + B_{t-1} \right) + p_{t} \left[\left(1 - \tau_{K} \right) \frac{r_{t}}{p_{t}} + \tau_{K} \delta_{K} \right] K_{t-1} - p_{t} I_{t} + \left(1 - \tau_{K} \right) \left(i_{t}^{D} \bar{M}_{t} + \Pi_{t} \right) + \left(1 - \tau_{L} \right) \left[w_{1,t} H_{1,t} + \bar{w}_{1,t} \left(1 - H_{1,t} \right) \right] + \rho_{1} \bar{G}_{t} + \left[\left(1 + \tau_{C} \right) p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\xi_{t}} \right) \right] + \left(1 - \tau_{K} \right) i_{s}^{D} \iota_{s} \right] C_{t}.$$
 (330)

The previous equation can be reorganized as:

$$F_{t} + B_{t} = R_{t} \left(F_{t-1} + B_{t-1}\right) + w_{1,t}H_{1,t} + \bar{w}_{1,t}\left(1 - H_{1,t}\right) + r_{t}K_{t-1} + \Pi_{t} + \varrho_{1}\bar{G}_{t} + \rho_{t}\left[1 + \varkappa_{t}^{d}\left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi}\frac{\xi_{t-1}}{\mathcal{E}_{t}}\right)\right]C_{t} + i_{t}^{D}\left(\bar{M}_{t} - \iota_{t}C_{t}\right) - p_{t}I_{t} + \left[1 + \varkappa_{t}^{d}\left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi}\frac{\xi_{t-1}}{\mathcal{E}_{t}}\right)\right]C_{t} + i_{t}^{D}\left(\bar{M}_{t} - \iota_{t}C_{t}\right) - p_{t}I_{t} + \left[1 + \varkappa_{t}\left[i_{t}F_{t-1} + (r_{t} - p_{t}\delta_{K})K_{t-1} + \Pi_{t} + i_{t}^{D}\left(\bar{M}_{t} - \iota_{t}C_{t}\right)\right] + \tau_{L}\left[w_{1,t}H_{1,t} + \bar{w}_{1,t}\left(1 - H_{1,t}\right)\right] + \tau_{C}p_{t}C_{t}\left[1 + \varkappa_{t}^{d}\left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi}\frac{\xi_{t-1}}{\mathcal{E}_{t}}\right)\right]\right]\right\}.$$
 (331)

Take now the definition of T_t into account:

$$F_{t} + B_{t} = (1 + i_{t}) F_{t-1} + R_{t} B_{t-1} + + (1 + \tilde{\tau}_{L}) w_{1,t} H_{1,t} + \bar{w}_{1,t} (1 - H_{1,t}) + r_{t} K_{t-1} + \Pi_{t} + \varrho_{1} \bar{G}_{t} - T_{t} + - p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_{t}} \right) \right] C_{t} + i_{t}^{D} \left(\bar{M}_{t} - \iota_{t} C_{t} \right) - p_{t} I_{t} + + t^{M} \left(1 - \Theta^{*} \right) N^{*} p_{M}^{*} x_{t}^{M} + \left(\tau_{L} + \tilde{\tau}_{L} \right) w_{2,t} H_{2,t}.$$
(332)

It's time now to take the government budget constraint into account:

$$F_{t} - (1 + i_{t}) F_{t-1} = (1 + \tilde{\tau}_{L}) w_{1,t} H_{1,t} + r_{t} K_{t-1} + \Pi_{t} + t^{M} (1 - \Theta^{*}) N^{*} p_{M}^{*} x_{t}^{M} + (1 - TR_{t}^{F}) (\tau_{L} + \tilde{\tau}_{L}) w_{2,t} H_{2,t} - INFR_INV_{t} - GCON_{t} + p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\xi_{t}} \right) \right] C_{t} + i_{t}^{D} \left(\bar{M}_{t} - \iota_{t} C_{t} \right) - p_{t} I_{t}.$$
 (333)

This takes us to the national accounting identity:

$$GDP_{t} = p_{t} \left[1 + \varkappa_{t}^{d} \left(\xi_{t} - \frac{1 - \delta_{D}}{\varphi} \frac{\xi_{t-1}}{\mathcal{E}_{t}} \right) \right] C_{t} + p_{t} I_{t} + GCON_{t} + INFR_INV_{t} + NX_{t}, \quad (334)$$

where:

$$GDP_{t} = (1 + \tilde{\tau}_{L}) w_{1,t} H_{1,t} + (1 + \tilde{\tau}_{L}) w_{2,t} H_{2,t} + r_{t} K_{t-1} + \Pi_{t} + i_{t}^{D} \left(\bar{M}_{t} - \iota_{t} C_{t} \right) + t^{M} \left(1 - \Theta^{*} \right) N^{*} p_{M}^{*} x_{t}^{M}, \quad (335)$$

and:

$$NX_t = CA_t - NFP_t, (336)$$

where CA_t , the current account surplus, and NFP_t , the net factor income from abroad, are equal to:

$$CA_t = F_t - F_{t-1}, (337)$$

$$NFP_{t} = i_{t}F_{t-1} - \left[1 - \tau_{L} + TR_{t}^{F}(\tau_{L} + \tilde{\tau}_{L})\right]w_{2,t}H_{2,t}.$$
(338)

Finally:

$$GNP_t = GDP_t + NFP_t. ag{339}$$

To compute the factor shares, assume that indirect taxation is attributed proportionally to capital and labor:

$$s_N GDP_t = (1 + \tilde{\tau}_L) w_{1,t} H_{1,t} + (1 + \tilde{\tau}_L) w_{2,t} H_{2,t} + s_N t^M (1 - \Theta^*) N^* p_M^* x_t^M.$$
(340)

This implies that:

$$s_N = \frac{(1+\tilde{\tau}_L) w_{1,t} H_{1,t} + (1+\tilde{\tau}_L) w_{2,t} H_{2,t}}{GDP_t - t^M (1-\Theta^*) N^* p_M^* x_t^M},$$
(341)

and consequently that:

$$s_{K} = \frac{r_{t}K_{t-1} + \Pi_{t} + i_{t}^{D}\left(\bar{M}_{t} - \iota_{t}C_{t}\right)}{GDP_{t} - t^{M}\left(1 - \Theta^{*}\right)N^{*}p_{M}^{*}x_{t}^{M}}.$$
(342)

Focusing on intratemporal trade in goods (produced in the intermediate-good sector, but considered final because either exported or imported):

$$IMP_{t}^{G} = (1 - \Theta^{*}) N^{*} (1 + t^{M}) p_{t,M}^{*} x_{t}^{M}, \qquad (343)$$

$$EXP_{t}^{G} = (1 - \Theta) Np_{t}^{F} y_{t}^{F} + (1 - \Theta^{*}) N^{*} p_{t}^{M,F} y_{t}^{M,F}, \qquad (344)$$

$$OPEN_t = \frac{IMP_t^G + EXP_t^G}{GDP_t}, \tag{345}$$

$$ToT_t = \frac{p_t^r}{p_t^M}. (346)$$

As far as trade in financial services is concerned:

$$NX_{t}^{SE} = (1 - \varrho^{FB}) i D_{t}^{FB} - i_{t}^{FB} D_{t}^{FB}.$$
(347)

9 Appendix B: LSM2 parameters and their calibrated value

As discussed in the main text, we can divide the LSM2 parameters into three groups according to the way we set their values. Here we provide additional details for the parameters of each group.

9.1 Parameter values based on other papers in the literature or theoretical considerations

- β : the subjective discount factor. We set this parameter to 0.995.
- v: the parameter related to the elasticity of substitution between consumption and dwellings in the utility function. We set the parameter in order to reproduce an elasticity of substitution equal to 1.5.
- σ : this parameter equals $1/\sigma^c$, where σ^c is the elasticity of intertemporal substitution. We assume logarithmic preferences, i.e. we set the parameter equal to unity.
- ξ_i the elasticity of the international interest rate with respect to the national debt/GDP ratio. Following Schmitt-Grohe and Uribe (2004), we set the parameter equal to 0.000742.
- δ_K : the depreciation rate of physical capital. Following Backus, Henriksen, and Storesletten (2008), we choose a value of 8.5%.
- ς : the elasticity of the adjustment cost with respect to the investment-capital ratio. Following Boldrin, Christiano, and Fisher (2001), we set the parameter equal to 1 1/0.23.
- Ξ_1 and Ξ_2 : the scale and intercept parameters of the adjustment cost function. Their values are $\Xi_1 = [\eta \gamma (1 \delta_K)]^{1-\varsigma}$ and $\Xi_2 = (1 1/\varsigma) [\eta \gamma (1 \delta_K)]$ to make the adjustment cost zero in the steady state.
- Θ : the share of non-traded domestic varieties. We set the parameter equal to 0.5.
- N: the number of available domestic differentiated intermediate goods. We set the value equal to 2.
- Θ^* : the share of traded foreign varieties (the share of importable varieties into Luxembourg). We choose a value equal to 0.5 for the sake of symmetry.
- N^* : the number of available foreign differentiated intermediate goods. We choose a value equal to 2, again for the sake of symmetry.
- ρ : the parameter capturing the increasing returns to variety. We assume no returns to variety in the benchmark parametrization, and set the parameter equal to 1.
- μ : the parameter related to the elasticity of substitution among intermediate goods. We set the parameter to obtain an elasticity equal to 1.2.
- ψ_K : the fraction of the rental bill the firms must finance in advance. We follow Christiano, Motto and Rostagno (2008, 2010) by setting its value to 0.92.

- ψ_L : the fraction of the wage bill the firms must finance in advance. We follow Christiano, Motto and Rostagno (2008, 2010) by setting its value to 0.92.
- ψ_i : the fixed cost to enter the market of intermediate good j. We choose a small value equal to 0.00001.
- θ_z : the relative bargaining power of the union for type z workers in the tradable and non-tradeable intermediate goods sectors. We choose a value equal to 0.35.
- θ_z^M : the relative bargaining power of the union for type z workers in the distribution sector. We choose a value equal to 0.27.
- θ_z^B : the relative bargaining power of the union for type z workers in the domestic and international banking systems. We choose a value equal to 0.65.
- P^* : the foreign aggregate price level. Normalized to unity.
- π_1 : the weights on imported foreign varieties in the production function of distributors of imported intermediate goods. We normalize its value to unity.
- TR_t^F : the percentage of total labour taxes on non-resident workers that is transferred back to non-resident workers. We choose a value equal to 0.6.
- δ_{INFR} : the depreciation rate of the stock of public infrastructure. The same reference as before suggests a value of 4.15%.
- ϑ : the persistence of core government expenditure. We choose a value equal to 0.9.
- α : the relative weight of physical capital in the CES production function. This parameter is strictly related to the capital share in output (actually, under a Cobb-Douglas specification, the two coincide). We set the parameter equal to 0.36, a standard value. The implied capital share in production under the benchmark parameterization lies around 39.2%.
- Λ : labour-augmenting productivity parameter. We normalize it to unity.
- a_1 : the parameter augmenting type-1 labour in the labour CES aggregator. It is normalized to unity.
- a_2 : the parameter augmenting type-2 labour in the labour CES aggregator. It is normalized to unity.
- κ : the parameter related to the elasticity of substitution between the two labour types in the CES labour aggregator. We set the value of the parameter in order to obtain an elasticity equal to 1.5.
- ϖ : the parameter related to the elasticity of TFP with respect to public infrastructure. We choose a value equal to 0.01.
- α^{CB} : the relative weight of physical capital in the production function of the domestic bank. For simplicity we set the parameter equal to the relative weight of physical capital in the manufacturing sectors, i.e. 0.36.

- ρ^{CB} : the required reserves coefficient for the domestic bank. We follow Christiano, Motto and Rostagno (2008, 2010) by setting its value to 0.02.
- σ^{FB} : the parameter related to the elasticity of foreign demand deposits to the interest rate paid by the international bank. We set its value to 1.5.
- α^{FB} : the relative weight of physical capital in the production function of the international bank. For simplicity we set the parameter equal to the relative weight of physical capital in the manufacturing sectors, i.e. 0.36.
- ρ^{FB} : the required reserves coefficient for the domestic bank. We follow Christiano, Motto and Rostagno (2008, 2010) by setting its value to 0.02.
- ρ : the persistence of the stochastic, persistent, but stationary component of productivity. We set the parameter equal to 0.95, a standard value in the literature.

9.2 Parameter values inferred from direct evidence on the value of the parameter:

- δ_D : the depreciation rate of durable goods. We average the BEA rates of depreciation for 'Durable goods owned by consumers' and set its value to 21.69%.
- φ : the individual survival rate, i.e. at the individual level, one minus the probability of dying at the end of the current period. Average life expectancy at birth in Luxembourg was 79.18 years in 2008 (CIA factbook): the survival rate that reproduces this outcome is 0.987.
- τ_K : the tax rate on profits and capital income. Because of data availability problems, the source *Taxation trends in the EU*, 2008, does not report an estimate of the average effective tax rate on capital. We take the average effective tax rate on corporate profits as a useful approximation, and set the parameter equal to 29.6%.
- τ_C : the tax rate on consumption (both durables and non-durables). We choose a value of 25.1%, taken from *Taxation trends in the EU*, European Commission, 2008. Note that the tax base for consumption tax includes non-durables consumption expenditure and the investment in durables.
- \overline{F} : The steady-state net foreign position relative to GDP. The average value of net foreign position was 95% and 75% of GDP at the end of 2007 and 2008, respectively (according to the bulletin of the Luxembourg Central Bank). Thus, we set the parameter to 0.85.
- τ_L : the tax rate on labour related income, paid by the employee. We follow again *Taxation trends in the EU*, 2008, and set the value to 20.1%. The figure has been obtained this way: the total average effective tax rate on labour equals 29,6%, but only 67,9% of this amount is paid by the employee. Hence, the average effective tax rate on labour paid by the employee becomes 20.1%.

- $\tilde{\tau}_L$: the social contribution rate on labour related income, paid by the employer. Given the previous result, we set the parameter to 9.5%.
- η : the population growth rate. We set the parameter equal to 1.012, since the current population growth rate in Luxembourg is 1.2% (data from CIA factbook, year 2008).
- t^F : the tariff on exported goods. As before, in 2007 88.2% of all exported goods were sold within the EEA and were exempt from tariffs. The remaining share of exported goods were subject to a tariff rate equal to 9%, which is the MA-OTRI in 2006 for the European Union. This is the ad-valorem equivalent of all tariff and non-tariff barriers that a country faces as an exporter. Thus, the effective tariff on exported goods is 1.062%, which is a weighted average of zero and 9%, where the weights are the respective export shares.
- t^M : the tariff on imported intermediate goods. The Overall Trade Restrictiveness Index in 2006 for the European Union equals 6.6%, as computed by the World Bank. This index is the ad-valorem equivalent of all tariff and non-tariff barriers that a country imposes on foreign imports. However, in 2007 94.5% of all imported goods originated from countries within the EEA and no tariff was applied in Luxembourg. Thus, the effective tariff on imported intermediate goods is 0.363%, which is a weighted average of zero and 6.6%, where the weights are the respective import shares.
- λ : the parameter related to the elasticity of substitution between capital and labour in the CES production function. Guarda (1997) estimates the elasticity to be 1.012 in the tradables sector. We set the value of the parameter in order to obtain the elasticity equal to 1.012.
- \varkappa_1 : the share of type-1 labour in the labour CES aggregator. We choose a value equal to 0.6 to reflect the fact that approximately 60% of the employed workforce is resident.
- γ: the rate of exogenous long-run technological progress. We set this parameter equal to 0.6%, which is the average TFP growth rate in Luxembourg over the 1995-2009 period, as reported in the Annual Report of the Luxembourg Central Bank (2006, p. 54).

9.3 Parameter values calibrated so the model matches observed ratios in the data:

- ϕ_C : the relative weight of durables consumption in the utility function. We calibrate the parameter in order to reproduce the share of non durable goods consumption expenditure from the final consumption expenditure of households equal to 0.884 (average annual share between 1995-2008). The implied value of ϕ_C is 0.8634.
- ϕ_D : the relative weight of non durables consumption in the utility function. We normalize the weights to sum up to unity hence its value equals to $1 \phi_C \phi_M = 0.1341$.
- ϕ_M : the relative weight of real money holdings kept for transaction services in the utility function. We calibrate the parameter in order to reproduce the ratio between nominal consumption of non-durables

and the currency stock in the households' hands (the velocity of money) equal to 7 (average between 2006-2009, OECD data). The implied value of ϕ_M is 0.0026.

- \overline{m} : the fixed individual endowment of money. We calibrate the parameter in order to reproduce the interest rate on demand deposits i^D equal to 1.94% (OECD-STAN data). The implied value is 0.1788.
- $\bar{\imath}$ the constant and exogenous long-run interest rate if the country settles down to a net foreign position equal to its steady-state value (interest rate risk premium is zero). We calibrate its value to match the observed net foreign position at 85% of GDP in Luxembourg (represented by \bar{F}). The implied value equals 2.074%.
- *REP*1 : replacement ratio of unemployment benefit for domestic workers, expressed as a share of the total gross income of employed domestic workers. We choose a value equal to 25.797% in order to replicate a 5% unemployment rate of type-1 workers.
- *REP*2 : replacement ratio of unemployment benefit for foreign workers, expressed as a share of the total gross income of employed domestic workers. We choose a value equal to 18.716% in order to replicate the ratio of type-1 to type-2 workers equal to 1.4238.
- Y^{*}: the foreign real output level. We calibrate it in order to reproduce the ratio between goods exports and GDP equal to 59.5% (average of annual values 2005-2008, data from CIA factbook 2008). The implied value is 0.4389.
- p_M^* : the foreign price level of imported intermediate goods. We calibrate its value in order to reproduce the ratio between goods imports and GDP ratio equal to 47.8% (average of annual values 2005-2008, data from CIA factbook 2008). The implied value is 0.4954.
- ρ_1 : the share of transfers to resident households in core (government) expenditure. We set the parameter equal to 42.557%, in order to make the model replicate the share of government transfers in **total** government expenditure (data from OECD annual national accounts, years 2003-2007).
- ρ_2 : the share of public investment in infrastructures in core (government) expenditure. We set the parameter equal to 11.489%, in order to make the model replicate the share of government investment in **total** government expenditure (data from OECD annual national accounts, years 2003-2007).
- d^{LR} : the parameter related to the long-run debt/GDP ratio. We calibrate the parameter in order to reproduce the observed debt/GDP ratio of Luxembourg at 0.069. The implied value for the parameter is 1.0008491.
- z^{CB} and z^{FB} : the sector specific productivity parameter of the domestic and the international banks. We restrict the productivity to be the same across the two banks and calibrate their value jointly to reproduce the observed ratio between interest rates on demand deposits and working capital equal to 2 (OECD-STAN data). The implied value is 55.9315.

- ξ^{CB} and ξ^{FB} : the parameter related to the share of demand deposits in the production function of the domestic and international banks. We restrict their values to be the same accross banks and jointly calibrate them to reproduce the observed ratio between demand deposits to households and to firms equals to 0.07 (OECD-STAN data). The implied value is 0.8958.
- D^* : We calibrate its value to reproduce the observed ratio between service trade balance and GDP equal to 46.82% (average of annual values 2005-2008, data from CIA factbook 2008). The implied value is 412.153.
- π_2 : the weight on capital and labor in the production function of imported intermediate goods distributors. We calibrate its value to make the model replicate the share of the transport sector value added from GDP (corrected for the estimated share of transport services that are devoted to international trade as in Ravn and Mazzenga (2004)) equal to 2.5% (average of annual values between 1995-2009, OECD-STAN data). The implied value is 8.041.

Table 1: Effects on selected variables of a 1% permanent increase in the replacement rate

A. Using LSM2

Aggregate	1y	2у	Зу	4y	5y	10y	20y
GDP							
Consumption							
Investment	-	-	-	-	-		
Net exports	+++						
Government deficit	+++	+++	+++	+++	+++	+++	+++
Capital stock	-	-	-	-	-	-	
Returns on capital							-
Profits							
Unemployment	+++	+++	+++	+++	+++	+++	+++
Employment, resident							
Employment, non residen							
Wages, resident	+	+	+	+	+	+	+
Wages, non resident	+	+	+	+	+	+	+
Total wages, resident							
Total wages, non resident							
Total assets	+	+	+	+	+	+	+
Total Factor Productivity	-	-	-	-	-	-	-

Horizon in years after the shock

B. Using LSM

	Horizon in years after the shock								
Aggregate	1у	2у	Зу	4y	5y	10y	20y		
GDP									
Consumption									
Investment	-	-	-	-	-				
Net exports									
Government deficit	+++	+++	+++	+++	+++	+++	+++		
Capital stock	-	-	-	-	-	-			
Returns on capital	-	-	-	-	-	-	-		
Profits									
Unemployment	+++	+++	+++	+++	+++	+++	+++		
Employment, resident									
Employment, non residen									
Wages, resident	+	+	+	+	+	+	-		
Wages, non resident	+	+	+	+	+	+	-		
Total wages, resident									
Total wages, non resident									
Total assets	+	+	+	+	+	+	-		
Total Factor Productivity	-	-	-	-	-	-	-		

Herizon in years often the sheel

Note: +, ++, and +++ indicate, respectively, an increase in the range 0-0.5%, 0.5-1% or larger than 1% with respect to the initial value. -, --, and --- indicate, respectively, a decrease in the range -0.5 - 0%, -1 -0.5% or smaller than -1% with respect to the initial value.

Table 2: Effects on selected variables of a 1% permanent decrease in MARKUP

A. Using LSM2

			Horizon in	years after	r the shock		
Aggregate	1y	2у	Зу	4y	5y	10y	20y
GDP	+	+	+	+	+	+	+
Consumption	++	++	++	++	++	++	++
Investment	+	+	+	+	+	+	+
Net exports		+	+	+	+	+	+
Government deficit							
Capital stock	+	+	+	+	+	+	+
Returns on capital	+	+	+	+	+	+	-
Profits	++	++	++	++	++	++	++
Unemployment	+++	+++	+++	+++	+++	+++	+++
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	+	+	+	+	+	+	+
Wages, non resident	+	+	+	+	+	+	+
Total wages, resident	+	+	+	+	+	+	+
Total wages, non resident	+	+	+	+	+	+	+
Total assets	-	-	-	-	-	-	-
Total Factor Productivity	+	+	+	+	+	+	+

B. Using LSM

	Horizon in years after the shock									
Aggregate	1у	2у	Зу	4y	5y	10y	20y			
GDP	+	+	+	+	+	+	+			
Consumption	++	++	++	++	++	++	++			
Investment	+	+	+	+	+	+	++			
Net exports	++	++	++	++	++	+	+			
Government deficit										
Capital stock	+	+	+	+	+	+	++			
Returns on capital	+	+	+	-	-	-	-			
Profits	++	++	++	++	++	++	++			
Unemployment	+	++	++	+++	+++	+++	+++			
Employment, resident	-	-	-	-	-	-	-			
Employment, non resident	-	-	-	-	-	-	-			
Wages, resident	+	+	+	+	+	+	+			
Wages, non resident	+	+	+	+	+	+	+			
Total wages, resident	+	+	+	+	+	+	+			
Total wages, non resident	+	+	+	+	+	+	+			
Total assets	-	-	-	-	-	-	+			
Total Factor Productivity	+	+	+	+	+	+	+			

Note: +, ++, and +++ indicate, respectively, an increase in the range 0-0.5%, 0.5-1% or larger than 1% with respect to the initial value. -, --, and - - - indicate, respectively, a decrease in the range -0.5 - 0%, -1 -0.5% or smaller than -1% with respect to the initial value.

Table 3a. Disaggregate effects of a 1% increase in FB productivity

	Horizon in years after the shock						
Sector / Variable	1v	2y	3v	4y	5v	10y	20y
·	,	,	,	,	,	,	,
International Pank							
Foreign demand denosits							
Foreign demaild deposits	+++	+++	+++	+++	+++	+++	+++
Interest on foreign demand deposits	+++	+++	+++	+++	+++	+++	+++
Capital stock	+++	+++	+++	+++	+++	+++	+++
Employment, resident	+++	+++	+++	+++	+++	+++	+++
Employment, non resident	+++	+++	+++	+++	+++	+++	+++
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	+++	+++	+++	+++	+++	+++	+++
Total wages, non resident	+++	+++	+++	+++	+++	+++	+++
Value added (GDP %)	+++	+++	+++	+++	+++	+++	+++
Domestic Bank							
Total demand deposits	+	+	+	+	+	+	++
Interest on demand deposits	+	+	+	+	+	+	+
Net interest rate on working capital	+	+	+	+	+	+	+
Capital stock	+	+	+	+	+	+	+
Employment, resident	++	++	++	++	++	++	++
Employment, non resident	++	++	++	++	++	++	++
Wages resident	_	_	_	_	_	-	_
Wages, non resident		-					
Total wages, resident	-	-	-	-	-	-	
Total wages, resident	- T	- T					11
Volue added (CDD %)	т	- T	т	т	- T	- T	
Value added (GDP %)	+	+	+	+	+	+	+
Non tradad							
Output							
Output Consistent at a sh							
Employment, resident							
Employment, non resident							
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident							
Total wages, non resident							
Profits							
Value added (GDP %)							
Traded							
Output							
Capital stock							
Employment, resident							
Employment, non resident							
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident							
Total wages, non resident							
Profits							
Value added (GDP %)							
Distributors							
Output							
Imported foreign varieties							
Canital stock							
Employment resident							
Employment, resident							
Wages resident							
wages, resident	-	-	-	-	-	-	-
wages, non resident	-	-	-	-	-	-	-
i otal wages, resident							
i otal wages, non resident							
Profits							
Value added (GDP %)							

Note: +, ++, and +++ indicate, respectively, an increase in the range 0-0.5%, 0.5-1% or larger than 1% with respect to the initial value. -, --, and --- indicate, respectively, a decrease in the range -0.5 - 0%, -1 -0.5% or smaller than -1% with respect to the initial value.

	Horizon in years after the shock								
Aggregate	1y	2у	Зу	4y	5y	10y	20y		
GDP	+	+	+	+	+	+	+		
Consumption	-	-	-	-	-	-	-		
Investment	+	+	+	+	+	+	+		
Net exports		+	+	+	++	++	++		
Government deficit									
Capital stock	+	+	+	+	+	+	+		
Returns on capital	+	+	+	+	+	+	+		
Profits									
Unemployment									
Employment, resident	++	++	++	++	++	++	++		
Employment, non resident	++	++	++	++	++	++	++		
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident	+	+	+	+	+	++	++		
Total wages, non resident	+	+	+	+	+	+	++		
Total assets	-	-	-	-	-	-	-		
Total Factor Productivity	+	+	+	+	+	+	+		

Table 3b. Aggregate effects of a 1% increase in FB productivity

Note: +, ++, and +++ indicate, respectively, an increase in the range 0-0.5%, 0.5-1% or larger than 1% with respect to the initial value. -, - -, and - - - indicate, respectively, a decrease in the range -0.5 - 0%, -1 -0.5% or smaller than -1% with respect to the initial value.
Table 4a. Disaggregate effects of a 1% increase in CB productivity

	Horizon in years after the shock								
Sector / Variable	1y	2y	Зy	4y	5y	10y	20y		
				-	-		-		
International Bank									
Foreign demand deposits	+	+	+	+	+	+	+		
Interest on foreign demand denosits	+	+	+	+	+	+	+		
Capital stock	-	- T	- T 	- T	- T	- T			
Capital Stock	- T	- T	- -	- T	- T	- T	- T		
Employment, resident	+	+	+	+	+	+	+		
Employment, non resident	+	+	+	+	+	+	+		
Wages, resident	+	+	+	+	+	+	+		
Wages, non resident	+	+	+	+	+	+	+		
Total wages, resident	+	+	+	+	+	+	+		
Total wages, non resident	+	+	+	+	+	+	+		
Value added (GDP %)	+	+	+	+	+	+	+		
Domestic Bank									
Total demand deposits	-	-	-	-	-	-	-		
Interest on demand deposits	-	-	-	-	-	-	-		
Net interest rate on working capital									
Capital stock									
Employment, resident									
Employment, non resident									
Wages resident	-	т.	<u>т</u>	1		1	-		
Wages, non resident									
Tatal wasan assident	+	+	+	+	+	+	+		
Total wages, resident									
lotal wages, non resident									
Value added (GDP %)									
Non traded									
Output	+	+	+	+	+	+	+		
Capital stock	+	+	+	+	+	+	+		
Employment, resident	+	+	+	+	+	+	+		
Employment, non resident	+	+	+	+	+	+	+		
Wages, resident	+	+	+	+	+	+	+		
Wages, non resident	+	+	+	+	+	+	+		
Total wages, resident	+	+	+	+	+	+	+		
Total wages, non resident	+	+	+	+	+	+	+		
Profits	+	+	+	+	+	+	+		
Value added (GDP %)	+	+	+	+	+	+	+		
					-				
Traded									
Output									
Capital stock	- T	- T	- -	- T	- T	- T	- T		
	+	+	+	+	+	+	+		
Employment, resident	+	+	+	+	+	+	+		
Employment, non resident	+	+	+	+	+	+	+		
Wages, resident	+	+	+	+	+	+	+		
Wages, non resident	+	+	+	+	+	+	+		
Total wages, resident	+	+	+	+	+	+	+		
Total wages, non resident	+	+	+	+	+	+	+		
Profits	+	+	+	+	+	+	+		
Value added (GDP %)	+	+	+	+	+	+	+		
Distributors									
Output	+	+	+	+	+	+	+		
Imported foreign varieties	+	+	+	+	+	+	+		
Capital stock	+	+	+	+	+	+	+		
Employment resident	, +	+	+	+	+	+	+		
Employment, non resident	- -	т 1	т 1	т 	т 1	т 1	, T		
Wages resident		т ,	т.	т	т	т			
wages, resident	+	+	+	+	+	+	+		
wages, non resident	+	+	+	+	+	+	+		
i otal wages, resident	+	+	+	+	+	+	+		
I otal wages, non resident	+	+	+	+	+	+	+		
Profits	+	+	+	+	+	+	+		
Value added (GDP %)	+	+	+	+	+	+	+		

	Horizon in years after the shock									
Aggregate	1у	2у	Зу	4y	5y	10y	20y			
GDP	+	+	+	+	+	+	+			
Consumption	+	+	+	+	+	+	+			
Investment	+	+	+	+	+	+	+			
Net exports		+	+	+	+	+	+			
Government deficit										
Capital stock	+	+	+	+	+	+	+			
Returns on capital	+	+	+	+	+	+	+			
Profits	+	+	+	+	+	+	+			
Unemployment	-	+	+	+	+	+	+			
Employment, resident	+	-	-	-	-	-	-			
Employment, non resident	+	+	+	-	-	-	-			
Wages, resident	+	+	+	+	+	+	+			
Wages, non resident	+	+	+	+	+	+	+			
Total wages, resident	+	+	+	+	+	+	+			
Total wages, non resident	+	+	+	+	+	+	+			
Total assets	-	-	-	-	-	-	-			
Total Factor Productivity	+	+	+	+	+	+	+			

Table 4b. Aggregate effects of a 1% increase in CB productivity

Table 5a. Disaggregate effects of a 1% decrease in D*

			Horizon in	years after	the shock		
Sector / Variable	1v	2v	3v	- 4y	5y	10y	20y
-				,	,		
International Bank							
Foreign demand denosits							
Interact on foreign domand donasite							
	-	-	-	-	-	-	-
Employment, resident							
Employment, non resident							
Wages, resident	+	+	+	+	+	+	+
Wages, non resident	+	+	+	+	+	+	+
Total wages, resident							
Total wages, non resident							
Value added (GDP %)							
Domestic Bank							
Total demand deposits	-	-	-	-	-	-	-
Interest on demand deposits	-	-	-	-	-	-	-
Net interest rate on working capital	-	-	-	-	-	-	-
Capital stock	_	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	_	_	=	_	_	_
Wages resident	۔ د	-	-	- ,L	-	-	-
wages, resident	+	+	+	+	+	+	+
wages, non resident	+	+	+	+	+	+	+
i otal wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Value added (GDP %)	-	-	-	-	-	-	-
Non traded							
Output	+++	+++	+++	+++	+++	+++	+++
Capital stock	+++	+++	+++	+++	+++	+++	+++
Employment, resident	+++	+++	+++	+++	+++	+++	+++
Employment, non resident	+++	+++	+++	+++	+++	+++	+++
Wages, resident	+	+	+	+	+	+	+
Wages, non resident	+	+	+	+	+	+	+
Total wages resident	+++	+++	+++	+++		+++	+++
Total wages, non resident							
Drofite							
Profiles	+++	+++	+++	+++	+++	+++	+++
value added (GDP %)	+++	+++	+++	+++	+++	+++	+++
Traded							
Output	++	++	++	++	++	++	++
Capital stock	+++	+++	+++	+++	+++	++	++
Employment, resident	++	++	++	++	++	++	++
Employment, non resident	++	++	++	++	++	++	++
Wages, resident	+	+	+	+	+	+	+
Wages, non resident	+	+	+	+	+	+	+
Total wages, resident	++	++	++	++	++	++	++
Total wages, non resident	++	++	++	++	++	++	++
Profits	++	++	++	++	++	++	++
Value added (GDP %)	++	++	++	++	++	++	++
Distributors							
Distributors							
Output	++	++	++	++	++	++	++
imported foreign varieties	++	++	++	++	++	++	++
Capital stock	+++	+++	+++	+++	+++	++	++
Employment, resident	++	++	++	++	++	++	++
Employment, non resident	++	++	++	++	++	++	++
Wages, resident	+	+	+	+	+	+	+
Wages, non resident	+	+	+	+	+	+	+
Total wages, resident	++	++	++	++	++	++	++
Total wages, non resident	++	++	++	++	++	++	++
Profits	++	++	++	++	++	++	++
Value added (GDP %)	++	++	++	++	++	++	++

	Horizon in years after the shock									
Aggregate	1y	2у	Зу	4y	5y	10y	20y			
GDP	-	-	-	-	-	-	-			
Consumption	+	+	+	+	+	+	+			
Investment	-	-	-	-	-	-	-			
Net exports	+++	-	-	-	-	-	-			
Government deficit	+++	+++	+++	+++	+++	+++	+++			
Capital stock	-	-	-	-	-	-	-			
Returns on capital	-	-	-	-	-	-	-			
Profits	++	++	++	++	++	++	++			
Unemployment	+++	+++	+++	+++	+++	+++	+++			
Employment, resident	-	-	-	-	-	-	-			
Employment, non resident	-	-	-	-	-	-	-			
Wages, resident	+	+	+	+	+	+	+			
Wages, non resident	+	+	+	+	+	+	+			
Total wages, resident	-	-	-	-	-	-	-			
Total wages, non resident	-	-	-	-	-	-	-			
Total assets	+	+	+	+	+	+	+			
Total Factor Productivity	-	-	-	-	-	-	-			

Table 5b. Aggregate effects of a 1% decrease in D*

Table 6a. Disaggregate effects of a 1% increase in interest rate

	Horizon in years after the shock								
Sector / Variable	1y	2y	Зу	4y	5y	10y	20y		
			· · · ·	· · · · ·	· · · · ·	· · ·	· · · ·		
International Bank									
Foreign demand deposits	+++	+++	+++	+++	+++	+++	+++		
Interest on foreign demand deposits	+++	+++	+++	+++	+++	+++	+++		
Capital stock	+++	+++	+++	+++	+++	+++	+++		
Employment, resident	+++	+++	+++	+++	+++	+++	+++		
Employment, non resident	+++	+++	+++	+++	+++	+++	+++		
Wages, resident	_	-	-	-	-	-	-		
Wages, non resident	_	_	_	_	_	_	_		
Total wages resident	+++	+++	+++	+++	+++	+++	+++		
Total wages, non resident	+++	+++	+++	+++	+++	+++	+++		
Value added (GDP %)	111	111	111	111	111	111	111		
Value added (GDF 76)									
Domestic Bank									
Total demand denosits									
Interest on domand deposits	++	++	++	++	++	++	++		
Net interest rate on working conital	-	-	-	+	+	+	+		
Net interest rate on working capital	-	-	-	-	-	+	+		
	+	+	+	+	+	+	+		
Employment, resident	++	++	++	++	++	++	++		
Employment, non resident	++	++	++	++	++	++	++		
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident	++	++	++	++	++	++	++		
Total wages, non resident	++	++	++	++	++	++	++		
Value added (GDP %)									
Non traded									
Output									
Capital stock									
Employment, resident									
Employment, non resident									
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident									
Total wages, non resident									
Profits									
Value added (GDP %)									
Traded									
Output									
Capital stock									
Employment, resident									
Employment, non resident									
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident									
Total wages, non resident									
Profits									
Value added (GDP %)									
Distributors									
Output									
Imported foreign varieties									
Capital stock									
Capital SLUCK									
Employment, resident									
Employment, non resident									
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident									
Total wages, non resident									
Profits									
Value added (GDP %)									

	Horizon in years after the shock										
Aggregate	1y	2у	Зу	4y	5y	10y	20y				
GDP	+	+	+	+	+	+	+				
Consumption											
Investment	+	+	+	+	+	+	+				
Net exports		+++	+++	+++	+++	+++	+++				
Government deficit											
Capital stock	+	+	+	+	+	+	+				
Returns on capital	++	++	++	++	++	+	+				
Profits											
Unemployment											
Employment, resident	++	++	++	++	++	++	++				
Employment, non resident	++	++	++	++	++	++	++				
Wages, resident	-	-	-	-	-	-	-				
Wages, non resident	-	-	-	-	-	-	-				
Total wages, resident	++	++	++	++	++	++	++				
Total wages, non resident	++	++	++	++	++	++	++				
Total assets	+	+	+	+	+	+	++				
Total Factor Productivity	+	+	+	+	+	+	+				

Table 6b. Aggregate effects of a 1% increase in interest rate

Table 7a. Disaggregate effects of a 1% point increase in FB reserves

			Horizon in	years after	the shock		
Sector / Variable	1y	2у	Зy	4y	5y	10y	20y
	,		,	,	,	,	,
International Bank							
Foreign demand denosits							
Interest on foreign demand denosits							
Consistent etc. etc. etc. etc. etc. etc. etc. etc							
Capital stock							
Employment, resident							
Employment, non resident							
Wages, resident	++	++	++	++	++	+	+
Wages, non resident	++	++	++	++	++	+	+
Total wages, resident							
Total wages, non resident							
Value added (GDP %)							
Domostic Bank							
Total damand damasita							
Total demand deposits							
Interest on demand deposits	-	-	-	-	-	-	-
Net interest rate on working capital	-	-	-	-	-	-	-
Capital stock	-	-	-	-	-	-	
Employment, resident							
Employment, non resident							
Wages, resident	++	++	++	++	++	+	+
Wages, non resident	++	++	++	++	++	+	+
Total wages, resident							
Total wages, resident							
Total wages, non resident							
Value added (GDP %)							
Non traded							
Output	+++	+++	+++	+++	+++	+++	+++
Capital stock	+++	+++	+++	+++	+++	+++	+++
Employment, resident	+++	+++	+++	+++	+++	+++	+++
Employment, non resident	+++	+++	+++	+++	+++	+++	+++
Wages, resident	++	++	++	++	++	+	+
Wages, non resident	++	++	++	++	++	+	+
Total wages, resident							
							+++
Total wages, non resident	+++	+++	+++	+++	+++	+++	+++
Profits	+++	+++	+++	+++	+++	+++	+++
Value added (GDP %)	+++	+++	+++	+++	+++	+++	+++
Traded							
Output	+++	+++	+++	+++	+++	+++	+++
Capital stock	+++	+++	+++	+++	+++	+++	+++
Employment, resident	+++	+++	+++	+++	+++	+++	+++
Employment, non resident	+++	+++	+++	+++	+++	+++	+++
Wages resident	++	++	++	++	++	+	+
Wages, non resident						_	
Total wages, resident							
	+++	+++	+++	+++	+++	+++	+++
lotal wages, non resident	+++	+++	+++	+++	+++	+++	+++
Profits	+++	+++	+++	+++	+++	+++	+++
Value added (GDP %)	+++	+++	+++	+++	+++	+++	+++
Distributors							
Output	+++	+++	+++	+++	+++	+++	+++
Imported foreign varieties	+++	+++	+++	+++	+++	+++	+++
Capital stock	+++	+++	+++	+++	+++	+++	+++
Employment resident	+++	+++	+++	+++	+++	+++	+++
Employment, resident	1.11	1.1.1	1.1.1	111	1.1.1	1.1.1	1.7.1
Employment, non resident	+++	+++	+++	+++	+++	+++	+++
wages, resident	++	++	++	++	++	+	+
Wages, non resident	++	++	++	++	++	+	+
Total wages, resident	+++	+++	+++	+++	+++	+++	+++
Total wages, non resident	+++	+++	+++	+++	+++	+++	+++
Profits	+++	+++	+++	+++	+++	+++	+++
Value added (GDP %)	+++	+++	+++	+++	+++	+++	+++

	Horizon in years after the shock									
Aggregate	1у	2у	Зу	4y	5y	10y	20y			
GDP	-	-	-	-	-	-	-			
Consumption	++	++	++	++	++	++	++			
Investment										
Net exports	+++									
Government deficit	+++	+++	+++	+++	+++	+++	+++			
Capital stock	-	-	-	-	-	-				
Returns on capital										
Profits	+++	+++	+++	+++	+++	+++	+++			
Unemployment	+++	+++	+++	+++	+++	+++	+++			
Employment, resident										
Employment, non resident										
Wages, resident	+	+	+	+	+	+	+			
Wages, non resident	+	+	+	+	+	+	+			
Total wages, resident										
Total wages, non resident										
Total assets	+	+	+	+	+	+	++			
Total Factor Productivity	-	-	-	-	-	-	-			

Table 7b. Aggregate effects of a 1% point increase in FB reserves

Table 8a. Disaggregate effects of a 1% point increase in CB reserves

			Horizon in	years after	r the shock		
Sector / Variable	1y	2у	Зу	4y	5y	10y	20y
			•		•		
International Bank							
Foreign demand deposits	-	-	-	-	-	-	-
Interest on foreign demand deposits	_	-	-	-	-	-	-
Capital stock			_		_	_	_
Employment resident	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Value added (GDP %)	-	-	-	-	-	-	-
Domestic Bank							
Total demand deposits	++	++	++	++	++	++	++
Interest on demand deposits	+++	+++	+++	+++	+++	+++	+++
Net interest rate on working capital	+++	+++	+++	+++	+++	+++	+++
Capital stock	+++	+++	+++	+++	+++	+++	+++
Employment, resident	+++	+++	+++	+++	+++	+++	+++
Employment, non resident	+++	+++	+++	+++	+++	+++	+++
Wages resident	_	-	-	-	-	-	_
Wages non resident		-	-	-	-	-	-
Tatal wasan and dant	-	-	-	-	-	-	-
Total wages, resident	+++	+++	+++	+++	+++	+++	+++
lotal wages, non resident	+++	+++	+++	+++	+++	+++	+++
Value added (GDP %)	+++	+++	+++	+++	+++	+++	+++
Non traded							
Output	-	-	-	-	-	-	-
Capital stock	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Profits	_	-	-	-	-	-	_
Value added (GDP %)							_
Value added (GDF 76)	_						
Tradad							
Tradea							
	-	-	-	-	-	-	-
Capital stock	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Profits	-	-	-	-	-	-	-
Value added (GDP %)	-	-	-	-	-	-	-
Distributors							
Output	_	-	-	-	-	-	_
Imported foreign variation	-	-	-	-	-	-	-
Capital stack	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Profits	-	-	-	-	-	-	-
Value added (GDP %)	+	+	+	+	+	+	-

	Horizon in years after the shock										
Aggregate	1у	2у	Зу	4y	5y	10y	20y				
GDP	-	-	-	-	-	-	-				
Consumption	-	-	-	-	-	-	-				
Investment	-	-	-	-	-	-	-				
Net exports	+++	-	-	-	-	-	-				
Government deficit	+++	+++	+++	+++	+++	+++	+++				
Capital stock	-	-	-	-	-	-	-				
Returns on capital	-	-	-	-	-	-	-				
Profits	-	-	-	-	-	-	-				
Unemployment	+	+	+	+	+	+	+				
Employment, resident	-	-	-	-	-	-	-				
Employment, non resident	-	-	-	-	-	-	-				
Wages, resident	-	-	-	-	-	-	-				
Wages, non resident	-	-	-	-	-	-	-				
Total wages, resident	-	-	-	-	-	-	-				
Total wages, non resident	-	-	-	-	-	-	-				
Total assets	+	+	+	+	+	+	+				
Total Factor Productivity	-	-	-	-	-	-	-				

Table 8b. Aggregate effects of a 1% point increase in CB reserves

Table 9a. Disaggregate effects of a 1% increase in working capital

			Horizon in	years afte	r the shock	1	
Sector / Variable	1y	2у	Зy	4y	5y	10y	20y
	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
International Bank							
Foreign demand denosits	_	-	-	-	-	_	_
Interest on foreign demand deposits	_	_	_	_	_	_	_
	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Value added (GDP %)	-	-	-	-	-	-	-
Domestic Bank							
Total demand deposits	++	++	++	++	++	++	++
Interest on demand deposits	++	++	++	++	++	++	++
Net interest rate on working capital	+	+	+	+	+	+	+
Capital stock	++	++	++	++	++	++	++
Employment, resident	++	++	++	++	++	++	+++
Employment, non resident	++	++	++	++	++	++	+++
Wages resident	1'T	(T	Υ T	(T	ГT	ſΤ	177
Wages, resident	-	-	-	-	-	-	-
vvages, non resident	-	-	-	-	-	-	-
i otal wages, resident	++	++	++	++	++	++	++
Total wages, non resident	++	++	++	++	++	++	++
Value added (GDP %)	++	++	++	++	++	++	++
Non traded							
Output	-	-	-	-	-	-	-
Capital stock	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	_	-	-	-	-	-	-
Total wages, resident	_	-	-	-	-	_	_
Total wages, non resident	_						_
Drofite							
Profiles	-	-	-	-	-	-	-
value added (GDP %)	-	-	-	-	-	-	-
Traded							
Output	-	-	-	-	-	-	-
Capital stock	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Profits	-	-	-	-	-	-	-
Value added (GDP %)	-	-	-	-	-	-	-
Distributors							
Output							
Uniput	-	-	-	-	-	-	-
imported foreign varieties	-	-	-	-	-	-	-
Capital stock	-	-	-	-	-	-	-
Employment, resident	-	-	-	-	-	-	-
Employment, non resident	-	-	-	-	-	-	-
Wages, resident	-	-	-	-	-	-	-
Wages, non resident	-	-	-	-	-	-	-
Total wages, resident	-	-	-	-	-	-	-
Total wages, non resident	-	-	-	-	-	-	-
Profits	-	-	-	-	-	-	-
Value added (GDP %)	-	-	-	-	-	-	-

	Horizon in years after the shock										
Aggregate	1у	2у	Зу	4y	5y	10y	20y				
GDP	-	-	-	-	-	-	-				
Consumption	-	-	-	-	-	-	-				
Investment	-	-	-	-	-	-	-				
Net exports	++	-	-	-	-	-	-				
Government deficit	+++	+++	+++	+++	+++	+++	+++				
Capital stock	-	-	-	-	-	-	-				
Returns on capital	-	-	-	-	-	-	-				
Profits	-	-	-	-	-	-	-				
Unemployment	+	+	+	+	-	-	-				
Employment, resident	-	-	-	-	+	+	+				
Employment, non resident	-	-	-	-	-	+	+				
Wages, resident	-	-	-	-	-	-	-				
Wages, non resident	-	-	-	-	-	-	-				
Total wages, resident	-	-	-	-	-	-	-				
Total wages, non resident	-	-	-	-	-	-	-				
Total assets	+	+	+	+	+	+	+				
Total Factor Productivity	-	-	-	-	-	-	-				

Table 9b. Aggregate effects of a 1% increase in working capital

Table 10a. Disaggregate effects of a 1% increase in FB productivity - Different bargaining

	Horizon in years after the shock								
Sector / Variable	1y	2y	Зy	4y	5y	10y	20y		
			· · · ·			· · ·	· · ·		
International Bank									
Foreign demand deposits	+++	+++	+++	+++	+++	+++	+++		
Interest on foreign demand deposits	+++	+++	+++	+++	+++	+++	+++		
Capital stock	+++	+++	+++	+++	+++	+++	+++		
Employment resident	+++	+++	+++	+++	+++	+++	+++		
Employment, resident	+++	+++	+++	+++	+++	+++	+++		
Wages resident		-	-	-	-	-	-		
Wages, nesident									
Total wagos, resident						-			
Total wages, resident	+++	+++	+++	+++	+++	+++	+++		
lotal wages, non resident	+++	+++	+++	+++	+++	+++	+++		
value added (GDP %)	+++	+++	+++	+++	+++	+++	+++		
De mentie Bergh									
Domestic Bank									
Total demand deposits	+	+	+	+	+	+	++		
Interest on demand deposits	+	+	+	+	+	+	+		
Net interest rate on working capital	+	+	+	+	+	+	+		
Capital stock	+	+	+	+	+	+	+		
Employment, resident	++	++	++	++	++	++	++		
Employment, non resident	++	++	++	++	++	++	++		
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident	+	+	+	+	+	++	++		
Total wages, non resident	+	+	+	+	+	++	++		
Value added (GDP %)	+	+	+	+	+	+	+		
Non traded									
Output									
Capital stock									
Employment, resident									
Employment, non resident									
Wages resident	_	-	-	-	-	-	_		
Wages, non resident	_								
Total wages resident									
Total wages, non resident									
Drofits									
Value added (CDD %)									
value added (GDP %)									
Traded									
Tradea									
Capital stock									
Employment, resident									
Employment, non resident									
Wages, resident	-	-	-	-	-	-	-		
Wages, non resident	-	-	-	-	-	-	-		
Total wages, resident									
Total wages, non resident									
Profits									
Value added (GDP %)									
Distributors									
Output									
Imported foreign varieties									
Capital stock									
Employment, resident									
Employment, non resident									
Wages, resident	-	-	-	-	-	-	-		
Wages non resident	_	-	-	-	-	-	_		
Total wages resident									
Total wages, non resident									
Drofite									
Value added (GDP %)									

		Horizon in years after the shock									
Aggregate	1у	2у	Зу	4y	5y	10y	20y				
GDP	-	+	+	+	+	+	+				
Consumption	-	-	-	-	-	-	-				
Investment	+	+	+	+	+	+	+				
Net exports		+	+	+	+	+	++				
Government deficit											
Capital stock	+	+	+	+	+	+	+				
Returns on capital	+	+	+	+	+	+	+				
Profits											
Unemployment											
Employment, resident	++	++	++	++	++	++	++				
Employment, non resident	++	++	++	++	++	++	++				
Wages, resident	-	-	-	-	-	-	+				
Wages, non resident	-	-	-	-	-	-	+				
Total wages, resident	+	+	+	+	+	++	++				
Total wages, non resident	+	+	+	+	+	++	++				
Total assets	-	-	-	-	-	-	-				
Total Factor Productivity	+	+	+	+	+	+	+				

Table 10b. Aggregate effects of a 1% increase in FB productivity - different bargaining

Table 11a. Disaggregate effects of a 1% increase in FB productivity - Lower union power

	Horizon in years after the shock							
Sector / Variable	1y	2у	Зу	4y	5y	10y	20y	
International Bank								
Foreign demand deposits	+++	+++	+++	+++	+++	+++	+++	
Interest on foreign demand deposits	+++	+++	+++	+++	+++	+++	+++	
Capital stock	+++	+++	+++	+++	+++	+++	+++	
Employment, resident	+++	+++	+++	+++	+++	+++	+++	
Employment, non resident	+++	+++	+++	+++	+++	+++	+++	
Wages resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident	+++	+++	+++	+++	+++	+++	+++	
Total wages, non resident	+++	+++	+++	+++	+++	+++	+++	
Value added (GDP %)	+++	+++	+++	+++	+++	+++	+++	
Domestic Bank								
Total demand deposits	+	+	+	+	+	+	++	
Interest on demand deposits						_		
Net interest rate on working capital	т 1	т .+	т .+	т "	т "	т _	T .L	
Capital stock		т 1	т 1	т 1	т 1	т 1	т 1	
Employment resident	،	т 44	T 44	т 44	т 4.4	ب بر	T 44	
Employment, resident		++ 	++ 	++ 	TT 4 1	TT 4 '	++	
Employment, non resident	++	++	++	++	++	++	++	
wages, resident	-	-	-	-	-	-	-	
wages, non resident	-	-	-	-	-	-	-	
i otal wages, resident	+	+	+	+	+	+	++	
Total wages, non resident	+	+	+	+	+	+	++	
Value added (GDP %)	+	+	+	+	+	+	+	
Non traded								
Output								
Capital stock								
Employment, resident								
Employment, non resident								
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident								
Total wages, non resident								
Profits								
Value added (GDP %)								
Traded								
Output								
Capital stock								
Employment, resident								
Employment, non resident								
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident								
Total wages, non resident								
Profits								
Value added (GDP %)								
Distributors								
Output								
Imported foreign varieties								
Capital stock								
Employment resident								
Employment, resident								
Wages resident								
Wages, resident	-	-	-	-	-	-	-	
voages, non resident	-	-	-	-	-	-	-	
Total wages, resident								
rotar wages, non resident								
Profits								
value added (GDP %)								

	Horizon in years after the shock							
Aggregate	1y	2у	Зу	4y	5y	10y	20y	
GDP	+	+	+	+	+	+	+	
Consumption	-	-	-	-	-	-	-	
Investment	+	+	+	+	+	+	+	
Net exports		++	++	++	++	++	++	
Government deficit								
Capital stock	+	+	+	+	+	+	+	
Returns on capital	+	+	+	+	+	+	+	
Profits								
Unemployment								
Employment, resident	++	++	++	++	++	++	++	
Employment, non resident	++	++	++	++	++	++	++	
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident	+	+	+	+	+	+	++	
Total wages, non resident	+	+	+	+	+	+	++	
Total assets	-	-	-	-	-	-	-	
Total Factor Productivity	+	+	+	+	+	+	+	

Table 11b. Aggregate effects of a 1% increase in FB productivity - lower union power

Table 12a. Disaggregate effects of a 1% increase in FB productivity - Lower share of banks in VA

	Horizon in years after the shock							
Sector / Variable	1y	2у	Зу	4y	5y	10y	20y	
International Bank								
Foreign demand deposits	+++	+++	+++	+++	+++	+++	+++	
Interest on foreign demand deposits	+++	+++	+++	+++	+++	+++	+++	
Capital stock	+++	+++	+++	+++	+++	+++	+++	
Employment, resident	+++	+++	+++	+++	+++	+++	+++	
Employment, non resident	+++	+++	+++	+++	+++	+++	+++	
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident	+++	+++	+++	+++	+++	+++	+++	
Total wages, non resident	+++	+++	+++	+++	+++	+++	+++	
	+++	+++	+++	+++	+++	+++	+++	
Domestic Bank								
Total demand deposits	++	++	++	++	++	++	++	
Interest on demand deposits	+	+	+	+	+	+	+	
Net interest rate on working capital	+	+	+	+	+	+	+	
Capital stock	+	+	+	+	+	+	+	
Employment, resident	++	++	++	++	++	++	++	
Employment, non resident	++	++	++	++	++	++	++	
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident	++	++	++	++	++	++	++	
Total wages, non resident	++	++	++	++	++	++	++	
Value added (GDP %)	++	++	++	++	++	++	++	
Non traded								
Output								
Capital stock								
Employment, resident								
Employment, non resident								
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident								
Total wages, non resident								
Profits								
Value added (GDP %)								
Traded								
Output								
Capital stock								
Employment, resident								
Employment, non resident								
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident								
Total wages, non resident								
Profits								
Value added (GDP %)								
Distributors								
Output								
Imported foreign varieties								
Capital stock								
Employment, resident								
Employment, non resident								
Wages, resident	-	-	-	-	-	-	-	
Wages, non resident	-	-	-	-	-	-	-	
Total wages, resident								
Total wages, non resident								
Profits								
Value added (GDP %)								

	Horizon in years after the shock								
Aggregate	1y	2у	Зу	4y	5y	10y	20y		
GDP	+	+	+	+	+	+	+		
Consumption	-	-	-	-	-	-	-		
Investment	+	+	+	+	++	++	++		
Net exports	++	++	++	++	++	++	++		
Government deficit									
Capital stock	+	+	+	+	+	++	++		
Returns on capital	++	+	+	+	+	+	+		
Profits									
Unemployment									
Employment, resident	++	++	++	++	++	++	++		
Employment, non resident	++	++	++	++	++	++	++		
Wages, resident	-	-	-	-	-	+	+		
Wages, non resident	-	-	-	-	-	+	+		
Total wages, resident	++	++	++	++	++	++	++		
Total wages, non resident	++	++	++	++	++	++	++		
Total assets	-	-	-	-	-	-	-		
Total Factor Productivity	+	+	+	+	+	+	+		

Table 12b. Aggregate effects of a 1% increase in FB productivity - lower share of banks in VA